Turkish Crystallographic Patterns: From Ancient To Present

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As far as I know, the term, Crystallographic Patterns (CPs), was first introduced by my PhD supervisor, the famous Azerbaijan Crystallographer Prof. Khudu Mamedov (1928-1988) [1, 2] (Fig. 1).



Fig. 1. Prof. Khudu S. Mamedov (Photograph by I. Hargittai)

Therefore, I would like to start my presentation by talking about him. He was born in 1927 in Marzili village of Karabag region, which is under the Armenian occupation nowadays. After graduating from the Geology Faculty of Azerbaijan State University, he started his PhD studies at the Institute os Crystallography of Academy of Sciences of USSR. His adviser was the well-known soviet crystallographer Prof. N. V. Belov, who advised his young assistant to resolve the structure of xonotlite [Ca₆Si₆O₁₇(OH)₂] and wollastonite [Ca₃Si₃O₉], which are the main constituents of cement silicates.

The structure of xonotlite mineral was completely a new type. Because of its originality, the Si_6O_{17} anion was different from the silicon-oxygen anions known until his time (Fig.2).

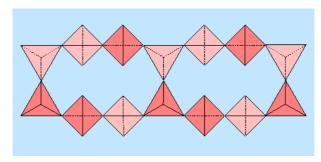


Fig.2. The structure of Si₆O₁₇ anion

With the discovery of this anion by Prof. Kh. Mamedov, it was found that the form and

chemical formulas of silicon-oxygen anions was depended on the size of cations outside of the tetrahedrons. After the finding of this anion, it became possible to see the big difference among the silicate structures resolved before and after the 2^{nd} World War.

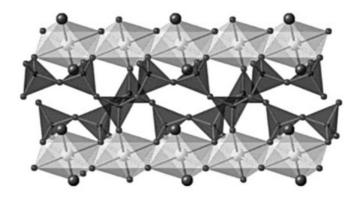


Fig.3. Structure of xonotlite

By resolving the xonotlite structure (Fig. 3) the structural chemistry of silicates was divided into two completely different chapter: the first chapter includes silicates containing small cations outside of tetrahedrons, while the second chapter includes the bigger cations outside of tetrahedrons. The first chapter of silicate crystal chemistry is remembered by the names of Bragg, Zahariasen, Taylor etc., while the second chapter is referred to as Belov and Mamedov.

After solving the structure of xonotlite, Kh. S. Mamedov resolved the structure an other cement silicate wollastonite with great success. He rejected the structural models suggested previously by M. Barnick [4] ve I. Ito [5] and introduced the completely new type structure and the structure of wollastonite was determined with high resolution [6] (Fig. 4). The explanation of the mechanism of hardening of cement became possible only after resolving the structure of xonotlite and wollastonite.

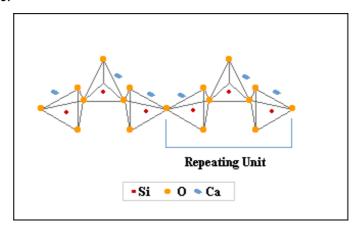


Fig.4. Crystal structure of wollastonite.

After defending his doctoral thesis in 1955, Kh. S. Mamedov returned to Baku, where he founded the Structural Chemistry Laboratory at the Institute of Inorganic and Physical

Chemistry of Academy of Sciences of Azerbaijan and continued his research on modelling and prediction of the calcium silicate and hydrosilicate structures. He published his monograph on the "Structural Chemistry of Calcium Silicates and Hydrosilicates" in 1960 in Azerbaijani. He has conducted postdoctoral studies in the J. Bernal's crystallography laboratory for six month in 1967. After defending his D. Sc. thesis, he was appointed as Professor in 1970. He founded the Crystal and Molecular Structure Laboratory at the Institute of Physics of Academy of Sciences of Azerbaijan in 1975. Until his death in 1988, he not only directed the research in these two laboratories, which included the synthesis and X-ray structural investigations of silicates, borates, zeolites, chalcogenides, semiconductor and coordination compounds, but also he prepared several young scientists in the field of crystallography and hence made great contribution to the national science of Azerbaijan.

Along with his intense scientific studies, he was attracted to art as well. His "Double Wing" book [7] on the analysis of the connection of art and science had great impact on us, university students in the 1970's. I was very lucky to have the chance to work in Mamedov's laboratories after I graduated from the faculty of chemistry of Azerbaijan State University in 1977. I, along with some of my colleagues, joined his art related studies, and as result of these studies, the book "Memory of Ornaments" [8] was published in 1981, which was about the Turkish ornaments having resemblance to crystalline structures. In 1986, on the request of Prof. I. Hargittai, he wrote a detailed paper on this subject, which he named "Crystallographic Patterns" [9] to be published in a book called "Symmetry" [10]. He has named these patterns as CPs due to their resemblance to the structure of crystals.

Among the first CPs that drew Kh. S. Mamedov's attention was the ornament on the Berde Tomb (Fig. 5).



Fig. 5. Fragment of Berde Tomb

This Tomb was built by Ahmed Ayyupoglu in 1322 AD. The words "Allah" were tiled on the cylindrical Tomb with colourful bricks. Professor of Crystallography Kh. S. Mamedov not only paid attention to the arrangement of those words on the surface of cylinder with plane group

P4, as in the arrangement of organic molecules in crystal, but also saw that there was much resemblance without exception. Those who study the history of architecture say that the master belongs to the Nakhchivan Architecture School. The founder of this school is Adjemi Nahchivani, the son of Ebubekir, who lived 850 years before.



Fig. 6. Mumine Hatun Tomb

Nearly everyone must have seen his Mumine Hatun Tomb he created, at least in the pictures (Fig. 6), which still stands with all its beauty and magnificence (the year 1186). The tomb in Karabaglar village (1335-1338) which belongs to Nahchivani School is shown in Fig.7.

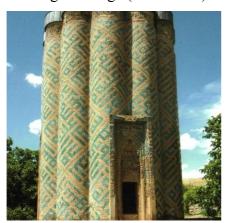


Fig. 7. Karabağlar Tomb

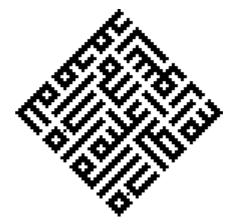


Fig. 8. Wall pattern of Karabağlar Tomb [11]

To ornament the surface of this tomb, a writing much more complex than that of Berde was chosen (*la ilaha illa Allah, Muhammad rasul Allah, salla Allah alaihi*) (Fig. 8), but the creativity principle of the pattern is still the same: the principle of creating crystallographic patterns. This square was also printed on 14th century Ilkhanid coins (Fig. 9) [11].



Fig. 9. 14th century Ilkhanid silver coin

While studying on the creation of such patterns chronologically and geographically, we saw that the CPs were widespread in Middle Asia, North Persia, Azerbaijan and Turkey. Excellent examples of CPs were created in Isfahan and Tabriz in the period of Seljuks and Safavids; and in the period of Timurids in Middle Asia. It is the fact that Timur took many masters to Samarkand from Azerbaijan to have it published and there exists information in Baburname about how Azerbaijan Turkish masters were taken to India.



Fig. 10. Shayh Safi Tomb

We see another variant of pattern in Berde Tomb as pronounced above in the Tomb of Shayh Safi in Erdebil (Fig.10). The same building technique, constructing of the same word in P4 plane group with only 45° difference in the arrangement of bricks. The pattern formed in the same plane group by the word "Allah" written a little different from that used in Berde Tomb is covering the walls of Mausoleum of Tughluq Temur in Chine (Fig. 11). The Mausoleum of Tughluq Temur is located in Huocheng, a town in the upper Ili River Valley of northern Xinjiang, near the border with Kazakhstan. It was ostensibly built in 1363 upon the death of Tughluq Temur Khan (1359-1363), the last descendant of Genghis Khan in the Chagatayid line.

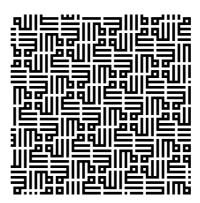
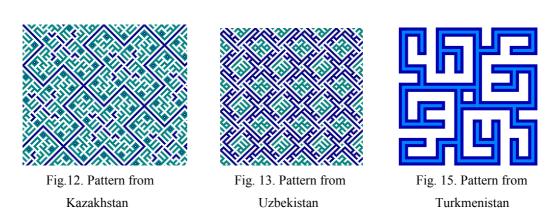


Fig. 11. Wall pattern of The Mausoleum of Tughluq Temur

In order for you to imagine or to understand how wide geography such patterns formed by kufic script, observed, I would like to give one example from each Middle Asian country. In Kazakistan: The memorial and shrine complex Ahmad Yasavi Mausoleum, which was built by Timur between 1389 and 1399, over the tomb of the Sufi saint Khoja Ahmad Yasavi (Fig.12). In Uzbekistan: The Aq Saray palace, which also was built by Timur in 1379-1396, at his hometown of Shahr-i Sabz. Its iwan pattern composed of the words of "Allah", "Muhammad" and "Ali" (Fig. 13). In Turkmenistan: The Shrine complex of Jamal al-Din at Anau, which was built between 1447-1457, during the reign of Abul-Kasim Babur (Fig. 14) [10].



Only one single sample is known in Anatolia, which was erected in the same technique. It is the Tomb (1473-1478) of Zeynel Bey, the son of Uzun Hasan, which was built with great mastery composed by the words of "Allah", "Muhammad" and "Ali" (Fig. 15). A more compact ornaments using the same words were created on the walls of Sheikh Lutfallah Mosque (Isfahan, 1603-1619) (Fig. 16).



Fig. 16. Ornament from Sheikh Lutfallah Mosque

Before I pass on to the other type ornaments I would like to remind you the Topkapı Architectural Design Scroll in Topkapı Palace Museum, which we known about from Prof. Gülru Necipoğlu's work. This is an example (Fig. 17) from the scroll, from the earliest known architectural scroll from the period "reflects the application of geometry in the Islamic tradition," according to David J. Roxburgh. In this ornament around the hexagonal outer band "Muhammad" is repeated six times; inside the central hexagon "Ali" is rotated three times [12].



Fig. 17. An example from Topkapı Scroll [12]

A similar, but more excellent ornament was observed in the Shirvanshah Palace in Baku. It is an architectural medallion with six "Ali", tree of which are written in shape of hollows in the stone, and the other tree on the just between the hollows (Fig. 18). "The old masters knew well enough that is was possible to achieve equality between the background and foreground; having done so they came up with antisymmetry, the black and white version of colored symmetry" [9].

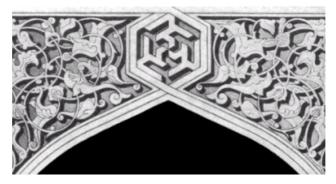


Fig. 18. Six "Ali" medalion from Shirvanshah Palace, Baku

Afterwards I have seen another version of six "Ali" medallion on the entrance of Sultanhani (Aksaray, Konya), which was built by Alaaddin Keykubat in 1229 (Fig.19) [13]. We have found a diversified simple of six "Ali" ornament in Jay Bonner's article on Islamic geometric ornaments (Fig. 20) [14].



Fig. 19. Six "Ali" medalion from Sultanhanı, Aksaray



Fig. 20. Six "Ali" medalion from the Masjid-i Jami, Varzana [14]

The other pattern drawing Kh. S. Mamedov's attention, was also another pattern on the Berde Tomb. As the Fig. 21 shows, this pattern was similar to structure of quartz mineral SiO_2 , at the same time, and crystals with chemical formula AX_2 in general. Since the Middle Ages, this pattern has been encountered in Anatolia. For example, the Green Tomb in Bursa is one of them (Fig. 22). The same pattern is frequently observed on the architectural monuments Middle Asia (Fig. 23).

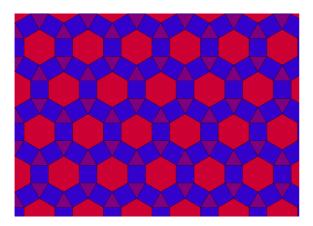


Fig. 21. Pattern from Berde Tomb



Fig. 22. The Green Tomb, Bursa



Fig. 23. Decoration from Bukhara, Uzbekistan

The pattern that embellish the Seljuk Palace in Ani (Kars) are shown in Fig. 24.



Fig. 24. Pattern from Seljuk Palace in Ani, Kars

The existence of crystals the structure of which resemble this pattern has been predicted by many crystallographers. The same pattern is seen in Konya Sahip Ata Mosque (1278) (Fig. 25) and Bursa Green Mosque (1419) (Fig. 26). This pattern is used widely in Middle Age miniatures.

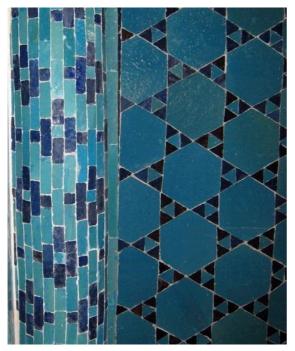


Fig.25. Pattern from Sahip Ata Mosque



Fig. 26. Pattern from Bursa Green Mosque

I want to draw your attention to formation of similar pattern in weaving a basket belonged to American Indians (Fig 27)[15]. When I met the same pattern on ceramic (Esmailabad/Karaj) dated 5000 BC exhibited in Azerbaijan Museum in Tabriz (Fig. 28) [16], I was surprised in a great extent. It means that this pattern is 7000 years old.



Fig. 27. Basket belonged to American Indians



Fig. 28. Pattern on ceramic from Azerbaijan Museum in Tabriz

As presenters before me talked about quascrystalline patterns [17], I want to give only one example of these patterns (Fig. 29) [18], I want to emphasize that above mentioned patterns have been created in the territories during Turkish hegemony ages.

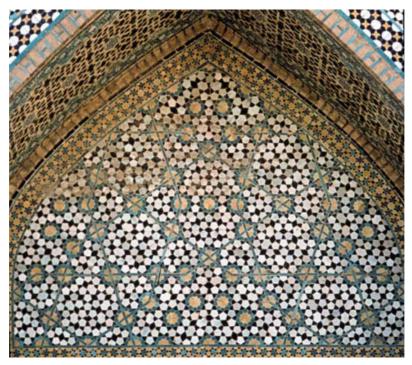


Fig. 29. Portal from the Darb-i Imam Shrine at Isfahan [18]

As Kh. S. Mamedov has recorded, "in all these countries words were used as design elements for creation of CPs, and along with words these patterns may be considered as different examples periodic tiling of the plane with polygons. And it is not surprising that they happened to be isomorphic with inorganic crystals which have been decoded in our own time... From the fact that words and phrases were widely used as pattern elements one might conclude that indeed the inclination toward geometric patterns is connected with the Islamic religion. Here arises a natural question: If this is really so, then such a trend should not be observed before Islam" [9]. Lots of people are trying to explain usage of CPs in Middle Ages commonly, as if Islam forbids drawing of human and animal depictions. Certainly art can be influenced by religion. For example, the main subject of European Renaissance is religion. But there are laws of arts itself. As art does not imitate the nature photographically, it does not base on religion and other fields. The fact of existence of countless human and animal depictions in our miniatures it needs not to evaluate such, influence of Islam in the creation of CPs. The miniature as you see in Fig. 30 belongs to a manuscript prepared for Shah Tahmasip, died CE 1576, of Firdawsi's Shah-Nama. This miniature which depicts the prophet of Islam and His relatives (Muhammad and Ali, with Ali's two sons Hasan and Huseyn), is also decorated by kufic scripts and geometrical ornaments [11].

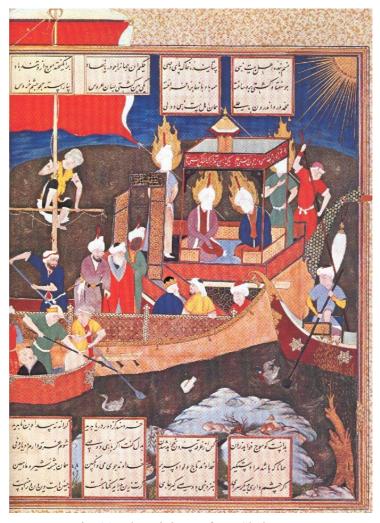


Fig. 30. The miniature from Shah-Nama

A review of pre-Islamic traditional fine art has shown that in Arabia, Persia, India, and partially in Afghanistan and southern parts of Central Asia, it was mainly anthropomorphic, while the Hun and Scythian area more characteristic is so-called Scythian beast style, which has features of CPs [8,9]. Scythian are known to have been the nomadic tribes dwelling on the wide plains between North of Black Sea and North Turkestan (many marks exist which impose the Turkish influence on them).

Fix attention to the "Four horses" figure (Fig. 31) [19] made by Reza Abbasi, who was Shah Abbas's (1587-1629) palace painter. Through using symmetry (the twofold axes), the number of the details (heads and legs) had been reduced twice. Such a usage of symmetry of the painter shows its painter's profound image about symmetry concept.

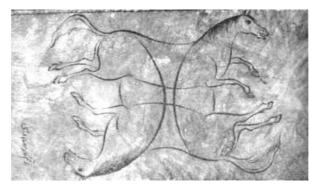


Fig. 31. "Four Horses" by Reza Abbasi.

According to Kh. S. Mamedov, for the nomads the subject of daily life always served as the symbol of the aesthetic sign, while in towns and cities there existed things which were specially intended to be considered as such symbol. Here also the subjects of daily life were sometimes used as the aesthetic sign symbols, but unlike the nomads, in this case the beauty was stuck on those subjects. You couldn't conceal the patterns unless you destroyed carpet, although you could change painting as much as you wished without destroying the brickwork of the wall on which it was painted. Thus, one might conclude that in the cities where the straight-line geometry was predominant the aesthetic signs were formed from the elements of reality itself, with nature playing the dominating role. This reverse symmetry between the aesthetic-sign attitude of the townsfolk and nomads can be clarified in the following schematic manner [9]:

	the surroundings	the art objects
Townsfolk	geometrical	natural
Nomads	natural	geometrical

A pattern of a ceramic in Fig. 32, dated 1000 years BC was found in West Siberia. The meaning of the details of this figure is not clear to us, but it is a typically crystallographic pattern. It is of great interest that we meet the same patterns in the contemporary artistic works of American Indians [20]. Pattern maker of Middle Ages created the same order using word written with kufic script instead of these elements.



Fig. 32. An ornament on ceramic, Tomsk, Russia



Fig.33. Example of Altaic culture

It is also possible to encounter such magnificent examples in ancient Altaic Culture. In Fig. 33 you see two birds of spirit are fighting against each other. There is no space here between the figures. The black bird is the background for the white one and the white bird is the background for the black one. The medallion with "Ali" (Fig. 18) is inscribed with the same principle. In the latter, six elements were used instead of two elements. As can be seen, the beginning of our pattern creation goes as far back as 1000 years BC and it is no wonder that Arabic letters were used as a prior element in the periods during which Islam was dominant.

We can see in Fig. 34 that such pattern creation was also widespread in 3000 years ago. Compare the ornament of the seals made of clay, which we found in the excavations in Sarıtepe near Kazakh city (Azerbaijan) with those of Fig. 35-38. We can also call these swastika. But, the first two figures were created with four "Ali", the others were created with four "Muhammad" and four "Allah" words.





Fig. 34. Seals from Kazakh, Azerbaijan

Fig. 35. Pattern on Bayazid-i Bistami Tomb

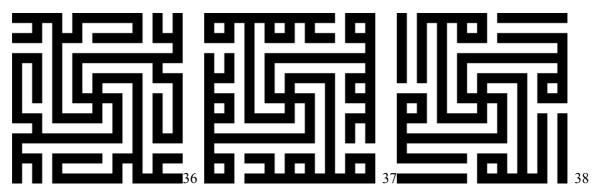


Fig. 35-37. Quadruple ornaments Ali, Muhammad, Allah from Friday Mosque of Isfahan

The similarity of these patterns implies that the way of creating patterns which existed in Azerbaijan from the times of bronze age has a close relation with the Siberian patterns. It is very interesting that we meet the patterns seen around Kazakh on the bronze axes dated BC XII-X centuries in Central Caucasus (Fig. 39). One of two seals (Fig. 40) founded in Altyn-depe (site in Turkmenistan, near Aşgabat, inhabited in the 3rd to 2nd millennia BC, abandoned around 1600 BC) [21] shows that roots of creation method of such ornaments are in the depth of History.



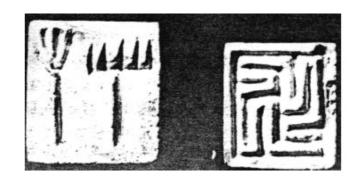


Fig. 39. The ornament on bronze axes, Ossetia, Rep. of Georgia

Fig. 40. Seals from Altyn-Depe, Turkmenistan

Excavation conducted in Kara Kum deserts showed that the history of CPs was older than Scythians. One of newest them belongs 3000 BC (Fig. 41) [22].

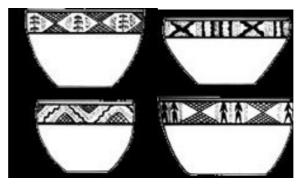


Fig. 41. Ornaments on ceramic from Kara Kum excavation, Turkmenistan

The last archeological survey revealed the existance of civilization older than Sumerian in Turkestan (4500 BC) [23]. The pattern on the stone seal dated 2300 BC, which was found by Prof. Fredrik Hiebert in Anau (Turkmenistan) is one of the ancient examples of CPs (Fig. 42) [24]. V. I. Sarianidi's idea that the roots of civilization is in Turkey is very interesting [25].

Even more interesting is the message that I met at discussion room in the http://www.turkotek.com web. In the message dated 04.02.2002 a man called Yon Bard compares the Anau pattern with carpet patterns from Turkmenistan (Fig. 43) [26]. When you look at the figure, you will see that there is no need any explanation. Announcement of Russian scientist Elena Tsareva about 6000 years history of Turkmen carpet is important in this point. [27]







Fig. 42. Stone seal from Anau, Turkmenistan Fig. 43. Comparison of Anau pattern with carpet pattern

From this point of view, the essay of famous archeologist James Mellart, reflecting the sameness of the wall pictures found in Çatalhöyük (Fig. 44) about 8000 years ago and the kilims of today's Anatolia (Fig. 45) [28] is very interesting. This subject has been dealt with in Mine Erbek's work called "Anatolian Motives from Çatalhöyük to the Present [29].



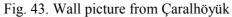




Fig. 44. Fragment of Anatolian kilim, Kastamonu

It is therefore of great importance to learn systematically about our certain patterns of BC. We should also remember what evidences are hidden below ground. Cultural history of the world's population shows that culture can be brought in and carried out. If a nation is inspired by the culture of another one, it keeps it alive and makes it a tool of creativity. The art and culture, which are brought by foreigners and invaders, are similar to foreign blood. Such blood does not develop in a new organism.

There are no works of art existing through centuries without not taking its root from creativity of people. That is, we suppose that the crystallographic patterns existing in works of art and rediscovered in Europe now are hidden in the creativity of the people. Are there traces of crystallographic patterns in the creativity of people? These traces are countless. Thumb through the books written about our carpets, and take another look at the patterns on carpets that are still kept at homes. We clearly see how the proper language of the patterns was kept in the creativity of the people in the countries where crystallographic patterns are widespread. This principle is also preserved available for music, folk creativity and poetry: less words, more meaning, plainness.

You can compare architectural pattern from Isfahan (Fig. 46) with ones from Baku and Tabriz carpets (Fig. 47, 48) [30].





Fig. 45. Ornament from Isfahan

Fig. 46. Kimim from Baku

Fig. 47. Kilim from Tabriz

Patterns in carpets and other waiving are called geometric. These patterns are the compositions made of geometric figures. Most of their elements are stylized pictures and probably bear a specific meaning.

The patterns made of plain geometric polygons are more widespread in architectural structures. As it is seen, patterns made from Arabic letter and alike complex figures are similar to structures of organic compounds. The patterns made of plain geometrical figures such as triangle, tetragon, hexagon are equal to structures of some inorganic compounds.

What is the reason of the similarity between crystal structures and crystallographic patterns. Of course, pattern creators could not have knowledge about matter formation then. The reason of the similarity may be accounted for by the way of some formation styles.

How are crystals formed from atoms, atom groups and molecules? In the same way as crystallographic patterns are formed by ornament elements. In other words, particles of materials should be placed so that maximum tightness can be achieved and a particle touches the maximum number of particles and particles be free in their positions. If such an ordered structure is not formed by the given particles, three-dimensional translation will not occur and the matter will turn into glass even if it hardens. That is crystal is not formed by arbitrary shaped molecules. Molecules should have complementary shapes. The figures in crystallographic patterns without background are complementary for one another. The plane is so covered with them that no space is seen among them, they complete one another. The molecules forming crystals should be such that they can fill the space of the crystal. Crystals form themselves by two, three or more shaped atom groups or molecules. In that case different shaped atom groups and molecules will have to

be complementary for one another. It is usually difficult to foretell the complementary quality of molecules and atom groups for the time being.

Crystallographic patterns are formed according to the complementary principles. The figures used as the ornament elements are placed on the plane such that there exists no space among them or the spaces are at equal sizes. In other words, pattern elements should be placed at a maximum tightness as molecules dispose themselves inside crystals. As seen, the way crystals are formed is the same with those of crystallographic patterns. Such is the case that if there is a resemblance between pattern elements and molecules or atom groups the illustration of the formation of a pattern and a crystal will be the same.

Of course, it is not possible to form crystallographic patterns from every kinds of shapes. It is necessary for the selected shape or shapes of a pattern to be complementary for one another. We know no rule to try whether they will be complementary for one another at the moment, but we are sure that it will be discovered in the future.

A bird eye' view on CP's shows that every art language has its own grammer. We will fail if we try to perceive one art's language through the grammar of another art.

How can it be possible to discover ancient art grammar? Of course by a thorough analysis of art formats studied by taking account general principles available for all art grammars.

In CP's such a goal has been achieved by using crystallography and its symmetry principles. Now we can make use of this field of science in the depiction and analysis of CP's. Another opportunity is to use the similarity between matter structures and patterns in education. This brings an aesthetic aspect to education. The invisible part of nature can be learned as a creativity of patterns. Every newly created CP is the structure plan of possible compounds, or vice versa, every newly resolved structure of inorganic compound makes the creation of new CP's possible. The familiarity with such ornaments and the ability to create them are important for solving compound structures. Similarity of patterns with some crystal structures enables us to reach the following conclusions: mankind may make use of the principles from which nature was created, and he may achieve a resemblance to the creation of nature in ideal; mankind may create nothing whose prototype does not exist in nature.

Kh. S. Mamedov's sudden death in 1988 prevented him from completing his studies on Turkish culture and art [31]. As Prof. Alan L. Mackay stated, "Mamedov was a great loss to us because he connected directly the age-old nomadic society and the modern trans-national scientific community and was able to explain the one to the other" [2]. I want to point out J. D. Bernal's words, which he wrote while presenting his book "The Origin of Life" to Kh. Mamedov: "To Dr. Mamedov whose contribution to crystal chemistry link the properties of life to those of mineral chemistry."

The CP's created by Kh. S. Mamedov were not published except a few of them during his lifetime. After a long period, two years ago, we published them in the book "Azerbaijani

tessellations" composed by Imameddin Amiraslan. I want to present some examples of them to you (Fig. 49-50).

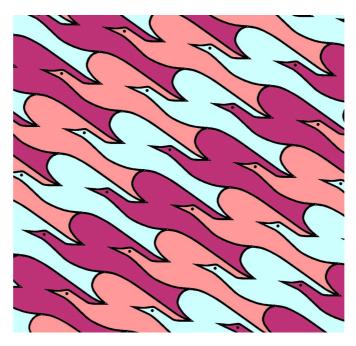


Fig. 49. "Birds" by Kh. S. Mamedov

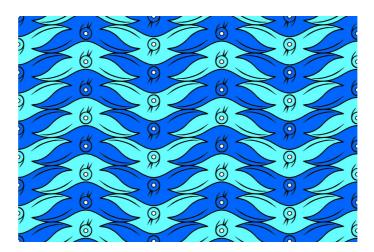


Fig. 50. "Gulls" by Kh. S. Mamedov

By Kh. S. Mamedov's influence, in 70's and 80's of the last century there had been a great interest in the CP's creation in Azerbaijan. Many tried to produce works of art in this field then. Imameddin Amiraslan is one of the most outstanding among them whom you listened to in this conference. Those who want to know his multi-directional creatin can have a look at the book mentioned above. I will give jast a few examples (Fig. 51-52) [32].

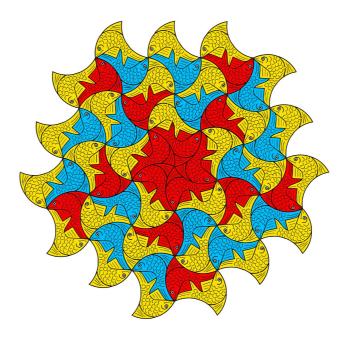


Fig. 51. "Fishes" by Imameddin Amiraslan

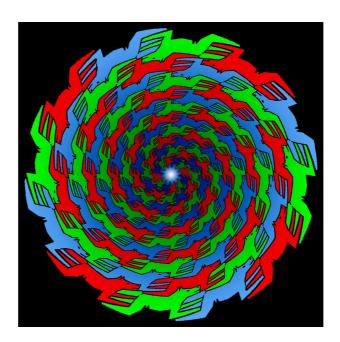


Fig. 52. "Wolfs" by Imameddin Amiraslan

You can see Kh. S. Mamedov's influence to modern Turkish painting in Fig. 53-54.



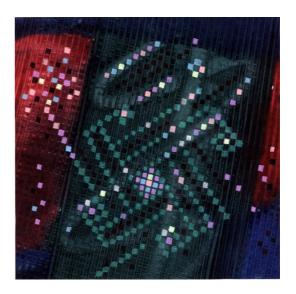


Fig. 53-54. Compositions by Ilham Enveroğlu

Through giving the illustration of some of CP's created by me (Fig. 55-56), I wanted to point that we have the chance to continue our distinguished art tradition.

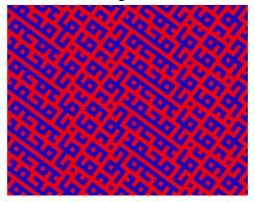


Fig. 55. "Khudu" by Hacali Necefoğlu

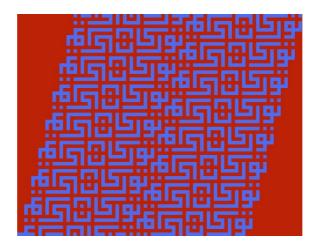


Fig. 56. "Türkan" by Hacali Necefoğlu

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