



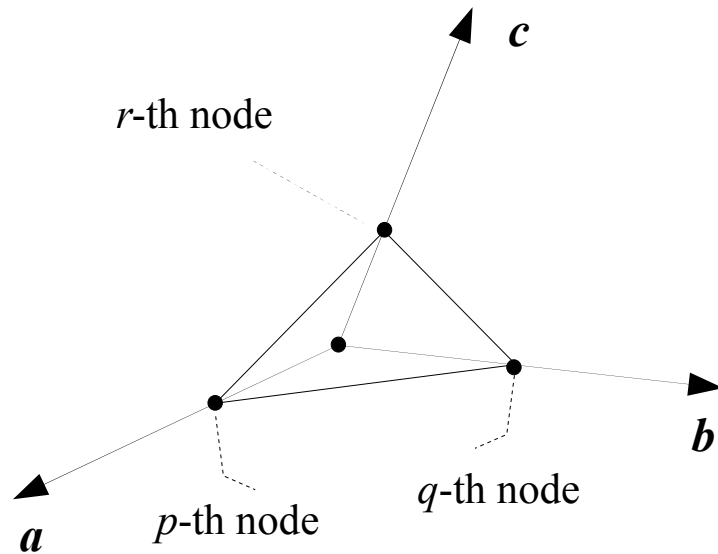
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Morphological symmetry. Introduction to the stereographic projection

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Lattice planes and Miller indices

Planes passing through lattice nodes are called “rational planes”



Parametric equation of the plane:

$$x/p + y/q + z/r = 1$$

$$(qr)x + (pr)y + (pq)z = pqr$$

$$hx + ky + lz = m$$

Making m variable, we obtain a *family* of lattice planes, (hkl) , where h , k and l are called the Miller indices.

First plane of the family (hkl) for $m = 1$

$$hx + ky + lz = 1$$

Intercepts of the first plane of the family (hkl) on the axes

$$p = pqr/qr = m/qr = 1/h$$

$$q = pqr/pr = m/pr = 1/k$$

$$r = pqr/pq = m/pq = 1/l$$

Example: family (112)

Intercepts of the first plane
of the family:

on a : 1/1

on b : 1/1

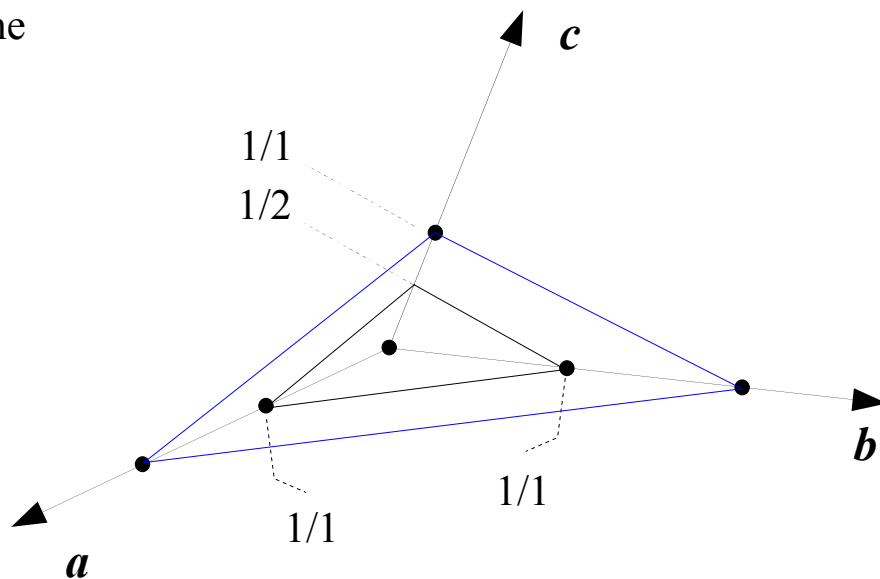
on c : 1/2

Intercepts of the second
plane of the family:

on a : 2/1

on b : 2/1

on c : 2/2



Example: family (326)

Intercepts of the first plane
of the family:

on a : $1/3$

on b : $1/2$

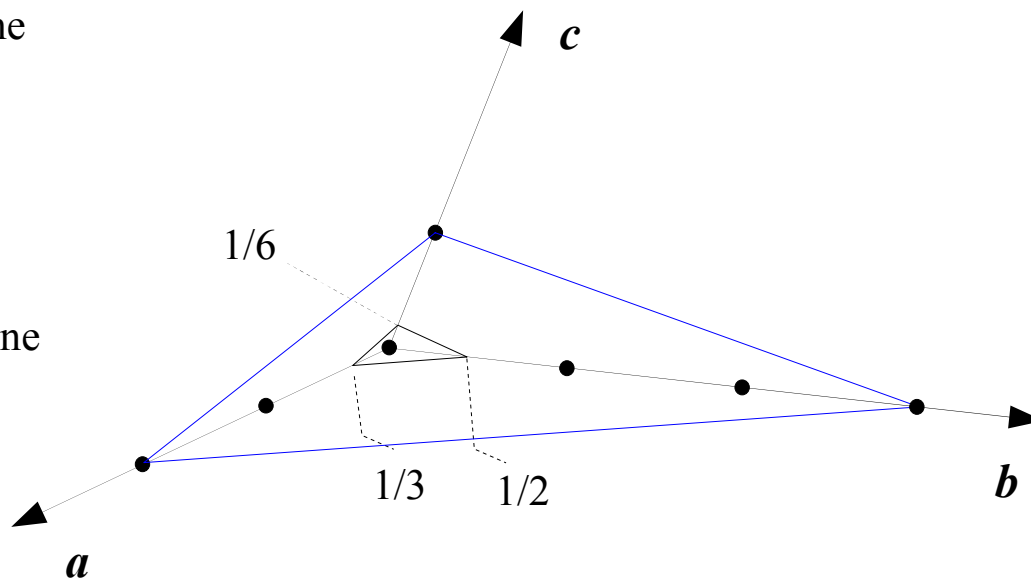
on c : $1/6$

Intercepts of the sixth plane
of the family:

on a : $6/3$

on b : $6/2$

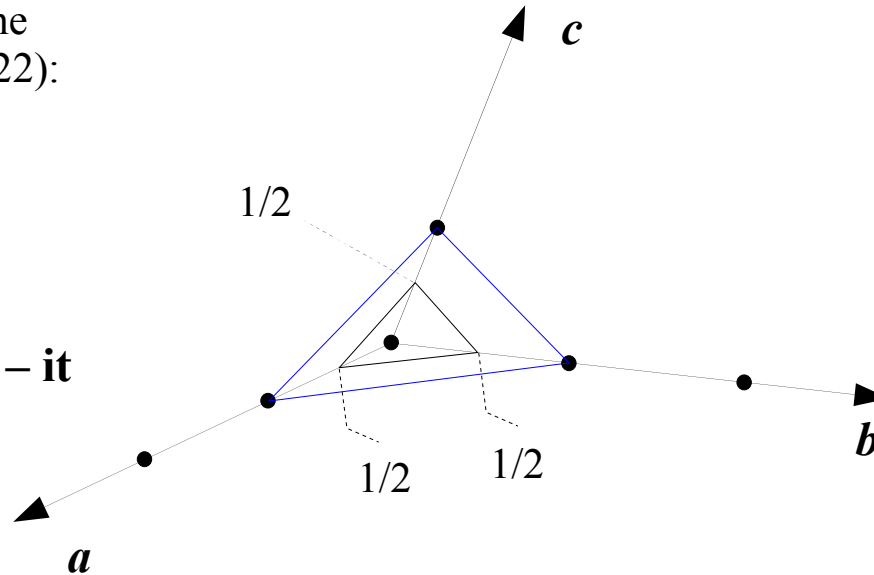
on c : $6/6$



Miller indices are coprime integers

Intercepts of the first plane
of a hypothetical family (222):
on a : $1/2$
on b : $1/2$
on c : $1/2$

**This plane does not pass
through any lattice node – it
is an *irrational* plane**



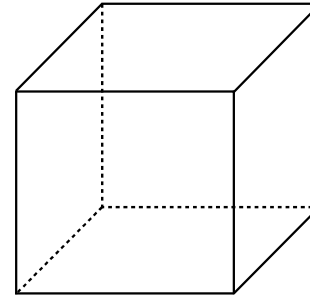
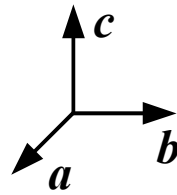
The first rational plane of
this family has intercepts:
on a : $1/1$
on b : $1/1$
on c : $1/1$

The family of planes has coprime indices (111)

The concept of form: set of faces equivalent by symmetry

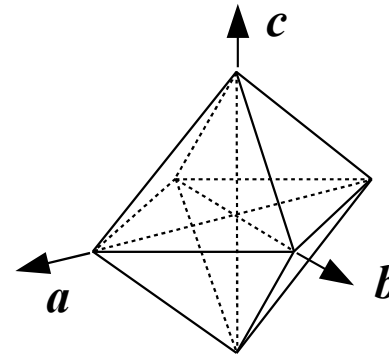
Example in the cubic crystal system

Form $\{100\}$: the cube



Multiplicity 6

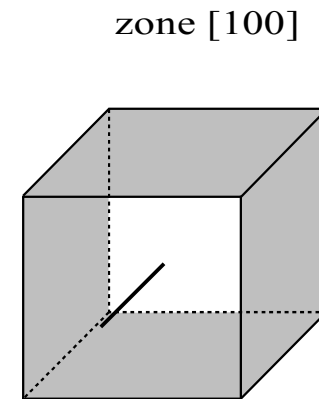
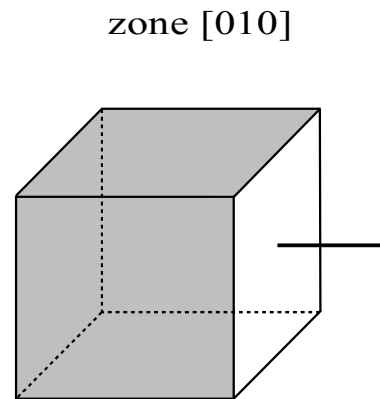
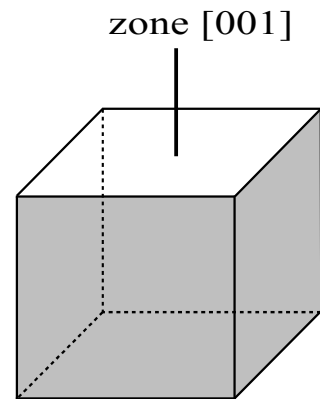
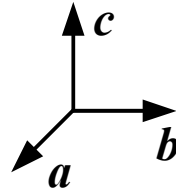
Form $\{111\}$: the octahedron



Multiplicity 8

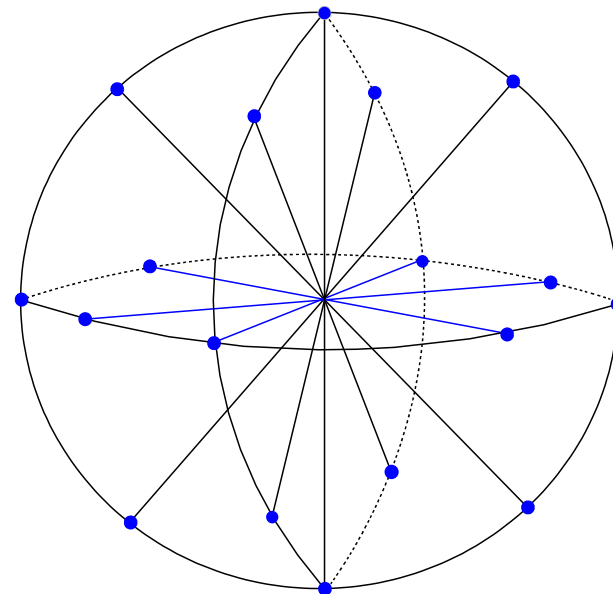
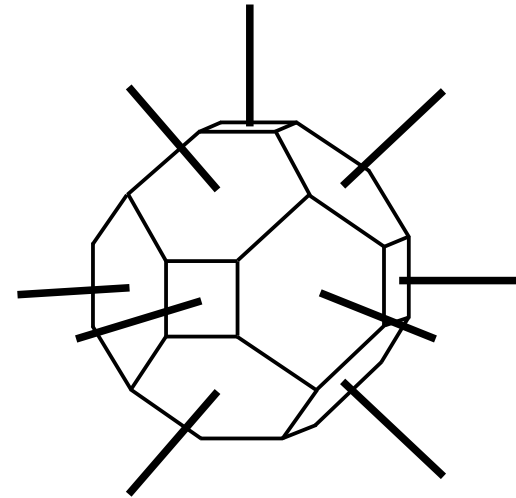
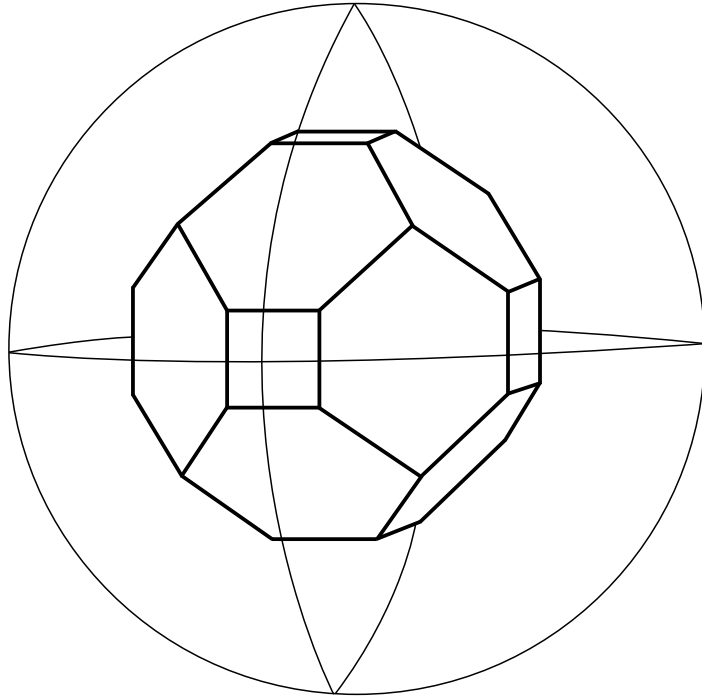
Zone: set of faces whose intersection is parallel to a same direction, called the zone axis:

Example in the cubic crystal system

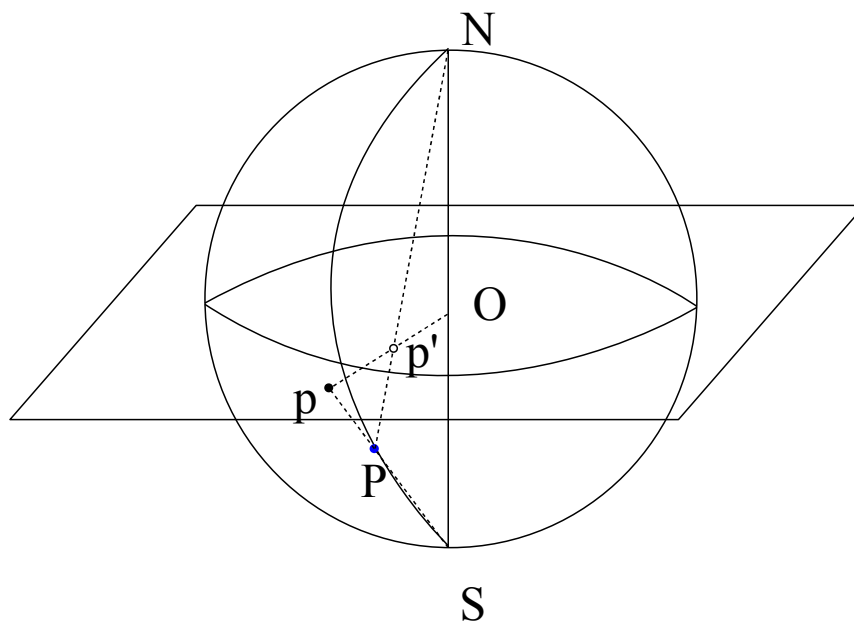
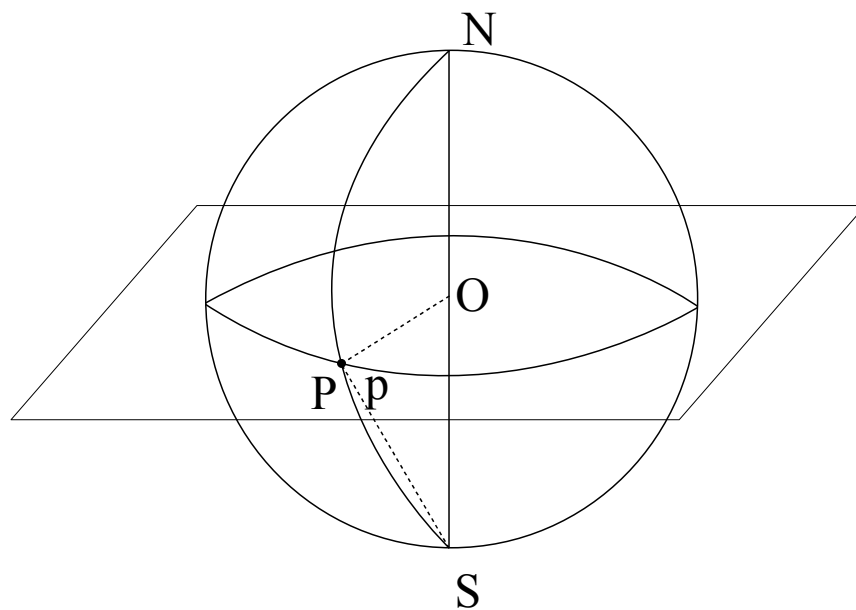
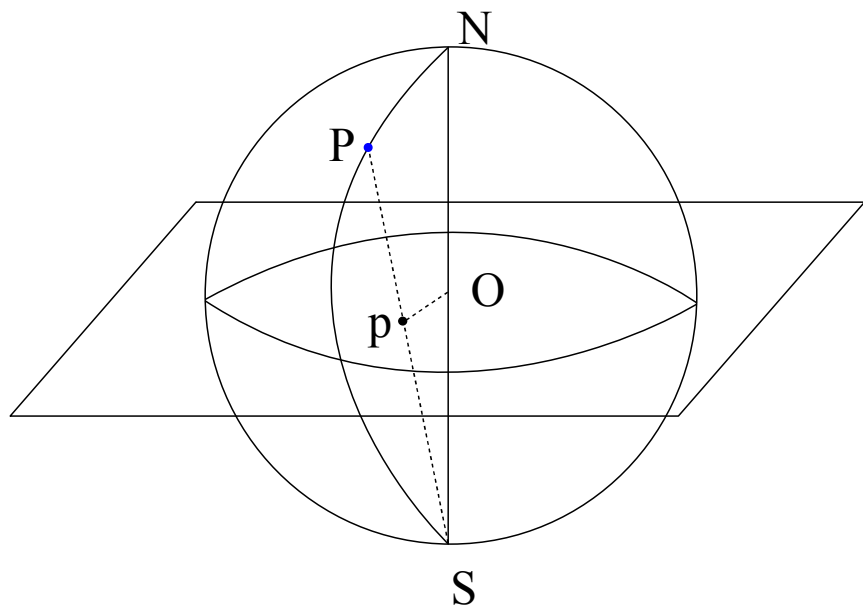


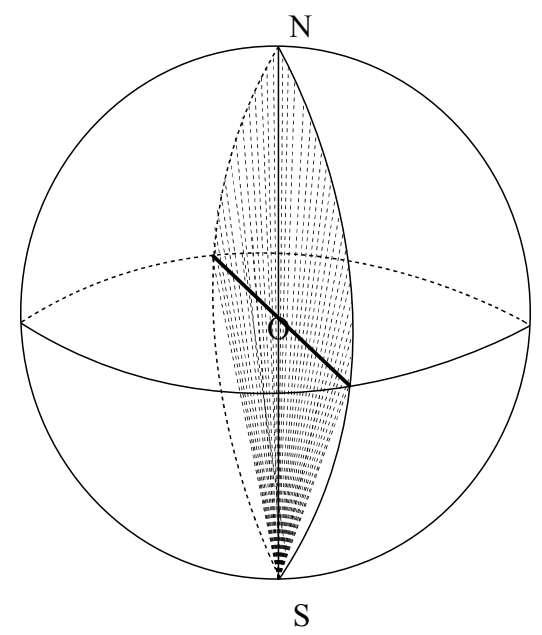
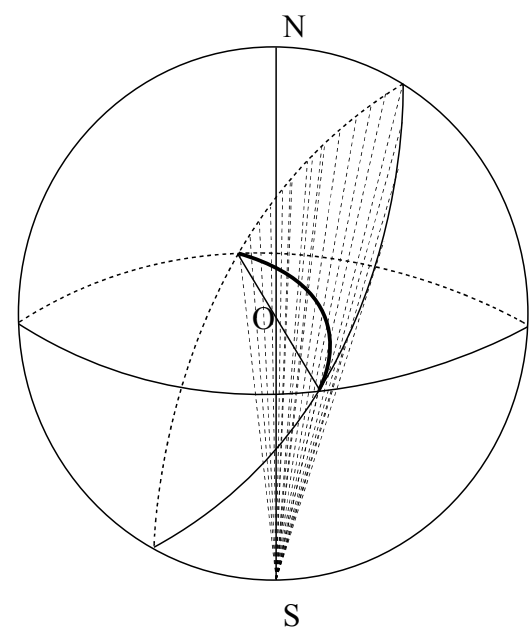
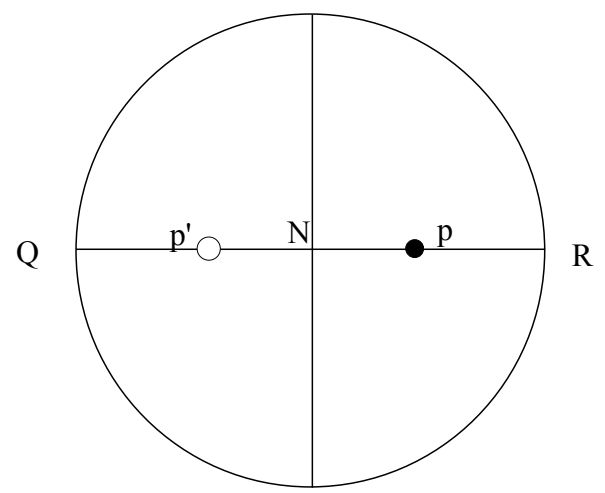
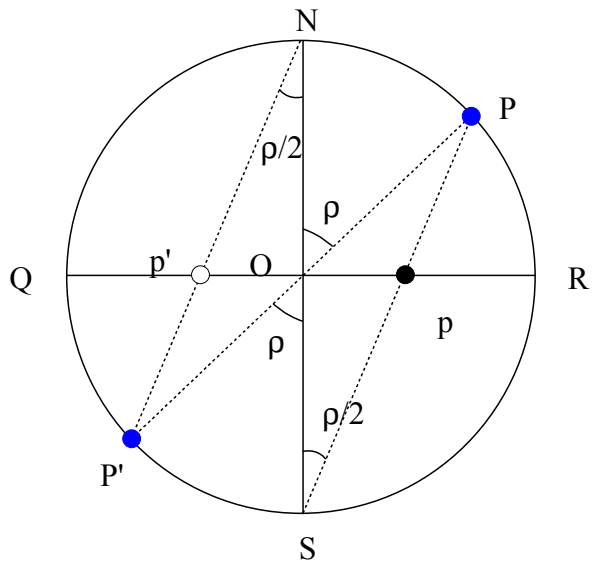
The stereographic projection: how to get rid of accidental morphological features of a crystal

Spherical projection and spherical poles

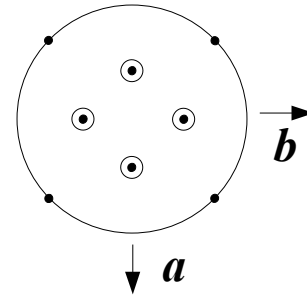
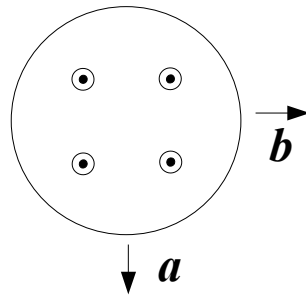
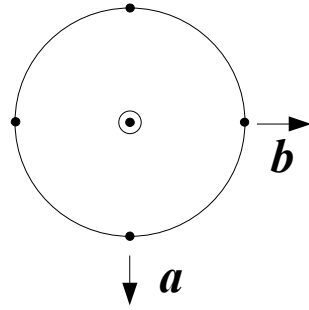


Building the stereographic projection: from the spherical poles (P) to the stereographic poles (p, p')





Example of decomposition of the morphology of a crystal



Cube

Octahedron

Dodecahedron

