Hierarchy of classification

Didactic material for the MaThCryst schools

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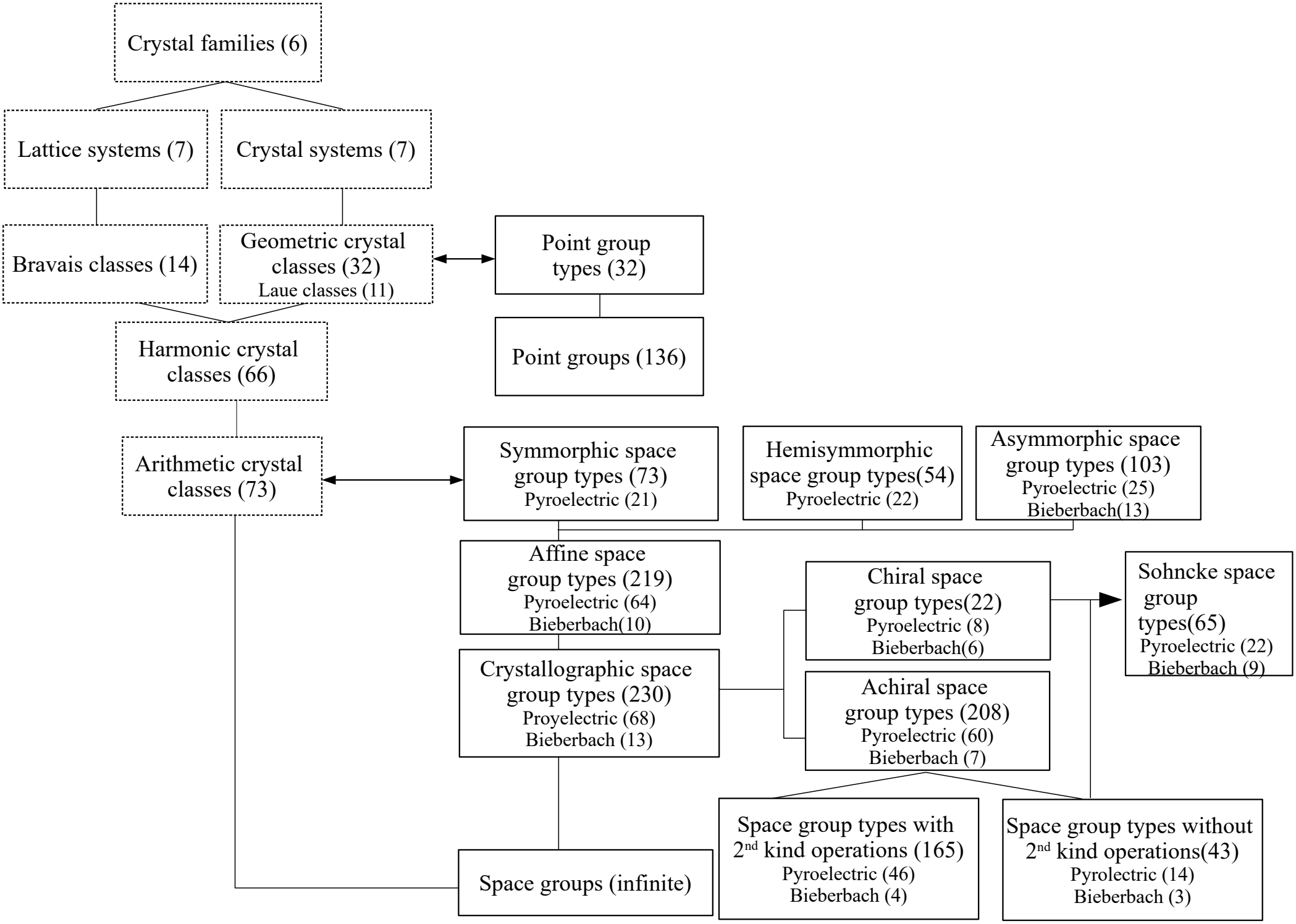
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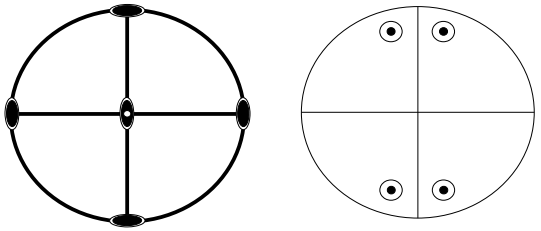
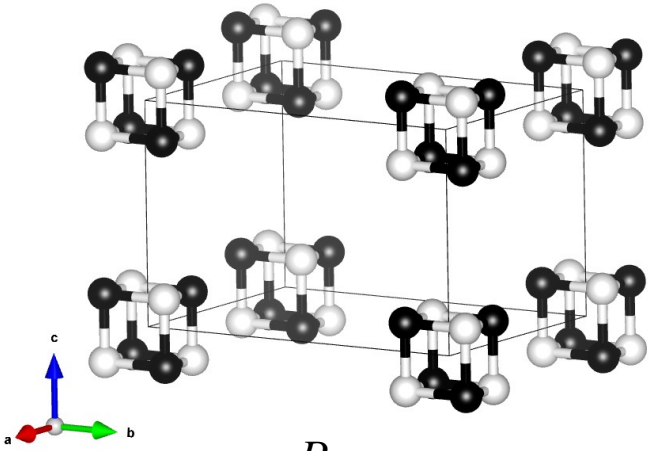
Point group



mmm

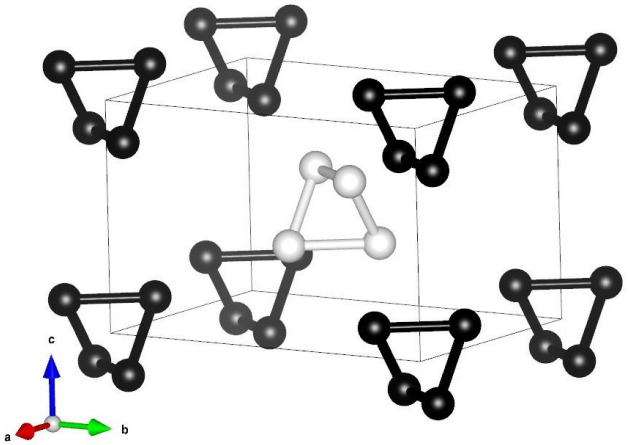
Symmorphic space group



Pmmm

site-symmetry group at the origin: *mmm*

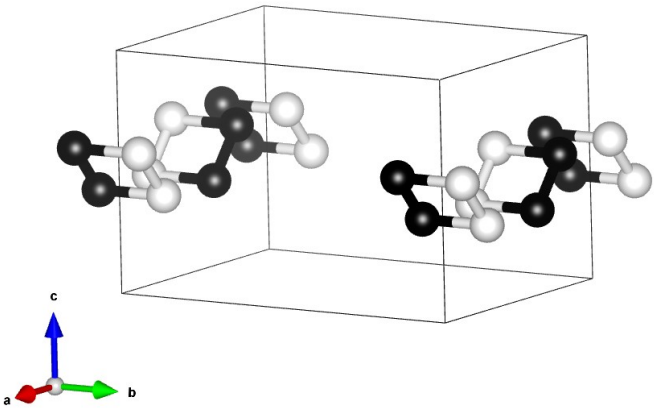
Hemisymmorphic space group



Pnnn

s.-s. group at the origin: 222

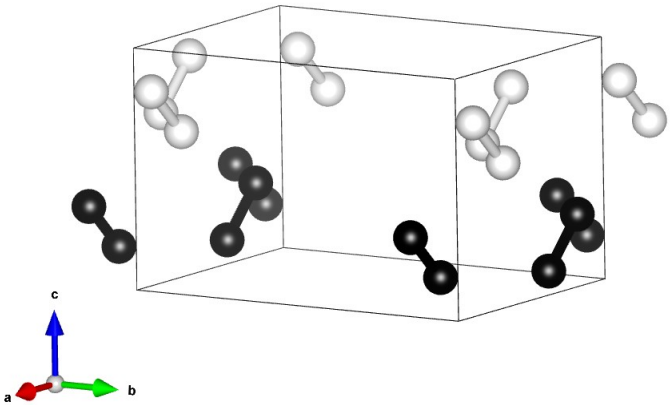
Asymmorphic space group



Pmma

s.-s. group at the origin: .2/*m*.

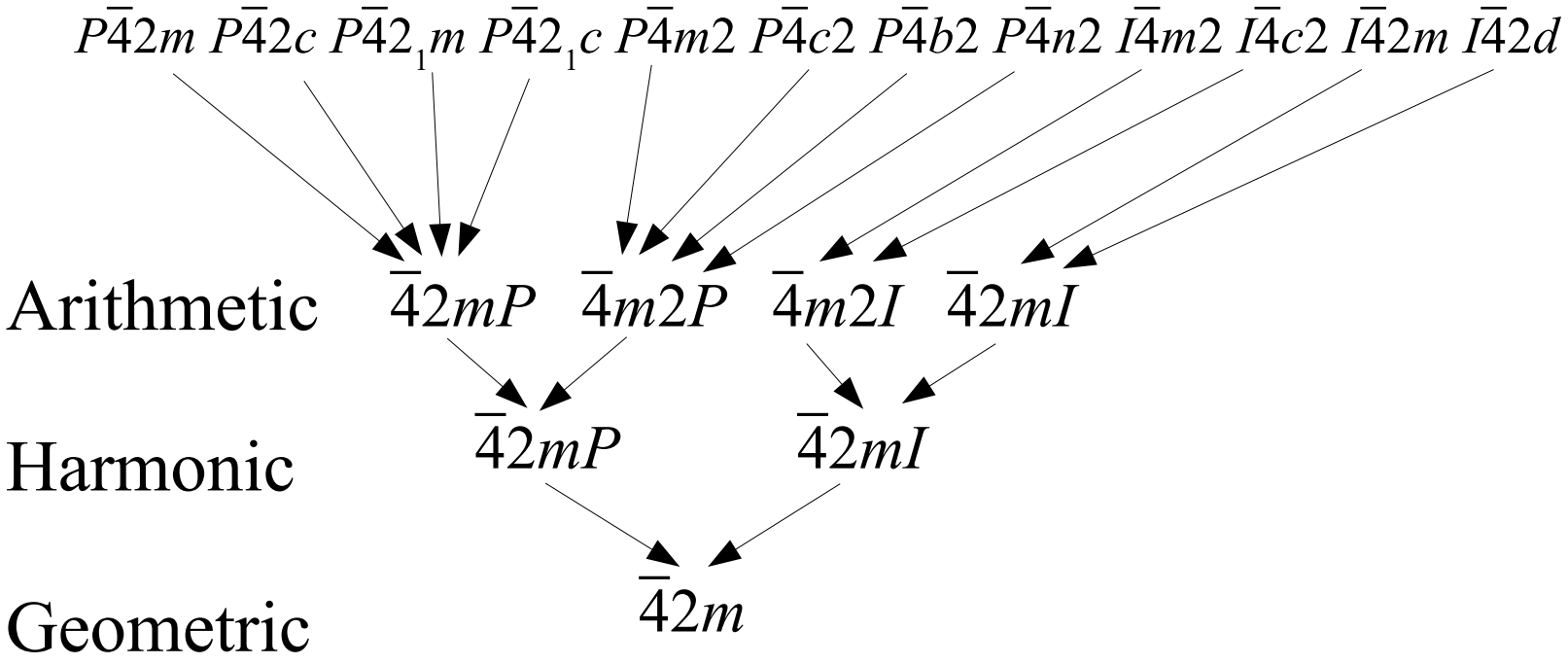
Asymmorphic space group



Pcca

s.-s. group at the origin: $\bar{1}$

Example of crystal classes hierarchy



A geometric crystal class which represents a full symmetry of a lattice is called a **holohedry**. Otherwise, a **merohedry**.

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Crystal systems

Space groups and crystal structures whose point group act of the same type of Bravais lattice belong to the same crystal system.

H, *G*: point groups $H \nrightarrow G$

If *G* acts on lattice, then *H* to acts on the same lattice

If *H* acts on lattice, *G* does ***not necessarily*** act on the same lattice

Ex. 1: $m\bar{3}m$ (*G*) acts on *cP*, *cI*, *cF*. 23, $m3$, 432, $\bar{4}3m$ (*H*) act on the same lattices

23 (*H*) acts on *cP*, *cI*, *cF*. $m\bar{3}m$ (*G*) act on the same lattices. Etc.

Crystals with point groups $m\bar{3}m$, $m3$, 432, $\bar{4}3m$, 23
all belong to the cubic crystal system.

Ex. 2: $6/mmm$ (*G*) acts on *hP*. 6, $\bar{6}$, $6/m$, 622, $6mm$, $\bar{6}2m$, 3, $\bar{3}$, 32, $3m$, $\bar{3}m$ (*H*) act on *hP* too.

$\bar{3}m$ (*H*) acts on *hP* and *hR* but $6/mmm$ (*G*) does not act on *hR*.

6 (*H*) acts on *hP*. $6/mmm$ (*G*) acts on *hP* too.

Crystals with point groups $6/mmm$, 6, $\bar{6}$, $6/m$, 622, $6mm$, $\bar{6}2m$ all belong to the hexagonal crystal system. Those with point groups 3, $\bar{3}$, 32, $3m$, $\bar{3}m$ belong to the trigonal crystal system.

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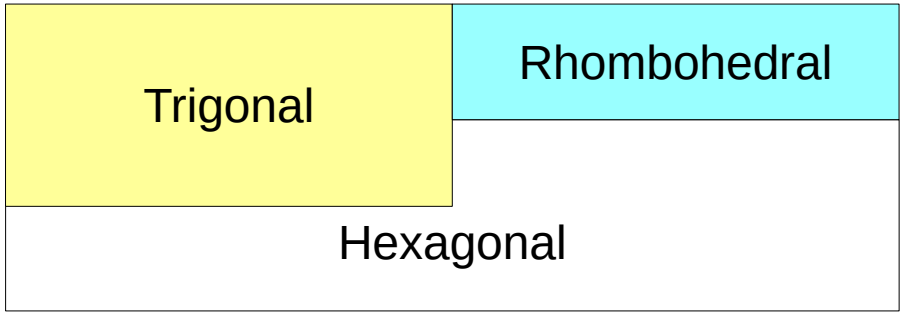
Lattice systems

Space groups and crystal structures which correspond to the same holohedry belong to the same lattice system.

Ex. 1: 23*P* and *m*3*P* correspond to the $m\bar{3}m$ holohedry and belong to the cubic lattice system.

Ex. 2: 32*P* and 622*P* correspond to the 6/*mmm* holohedry and belong to the hexagonal lattice system.

Ex. 3: 32*R* and 622*P* correspond to the different holohedries ($\bar{3}m$ and 6/*mmm*). 32*R* belong to the rhombohedral lattice system, whereas 622 belongs to the hexagonal lattice system.



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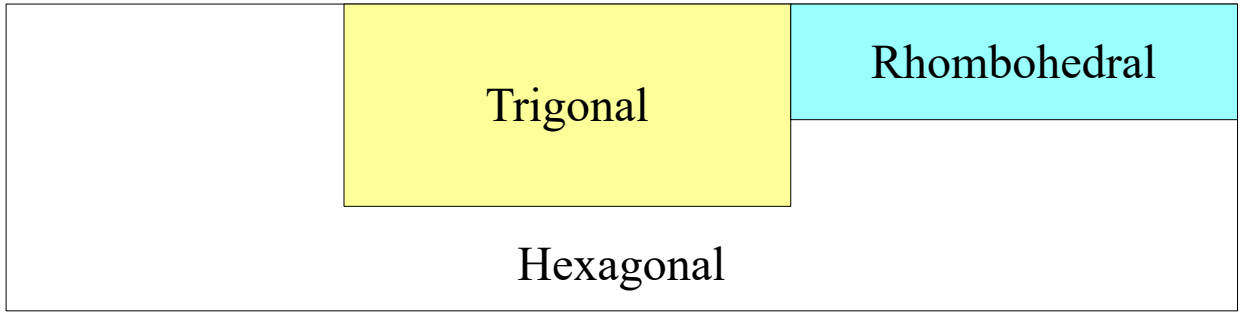
Crystal families

Space groups and crystal structures whose lattices have the same number of free parameters belong to the same crystal family if the corresponding point groups are in group-subgroup relation.

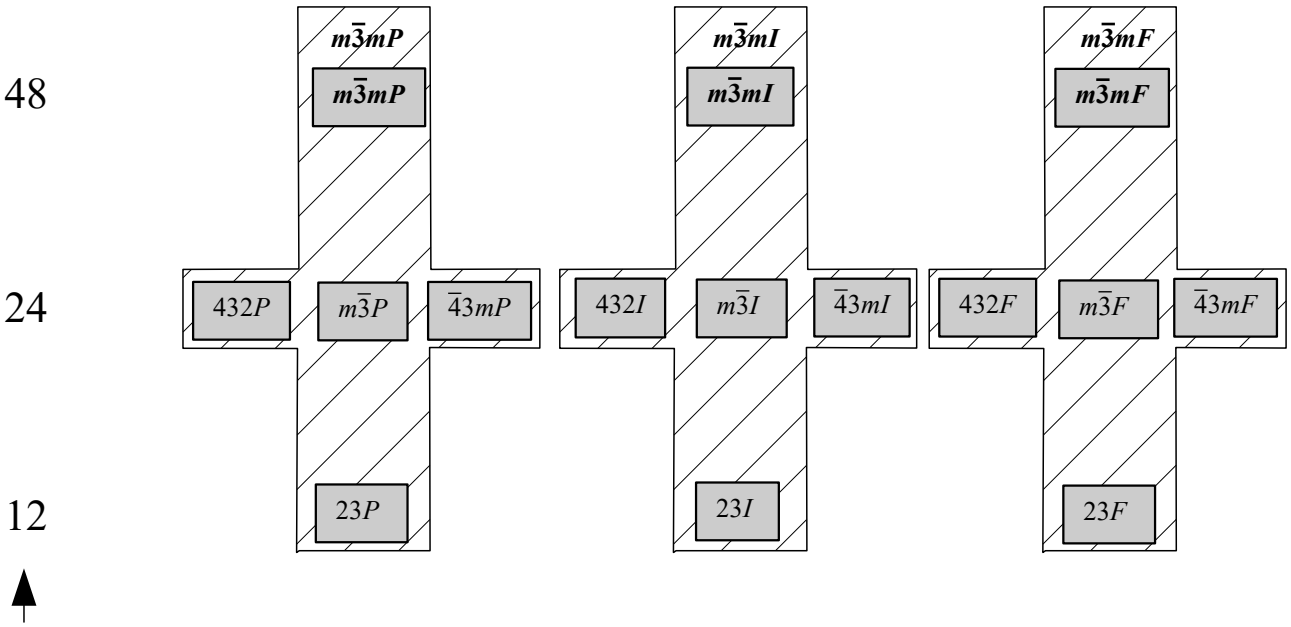
Ex. 1: Two crystals with holohedries 4/*mmm* and 6/*mmm* have lattices with two free parameters (*a* and *c*). However, 4/*mmm* and 6/*mmm* are not in group-subgroup relation and thus the two crystals belong to different crystal families (tetragonal and hexagonal).

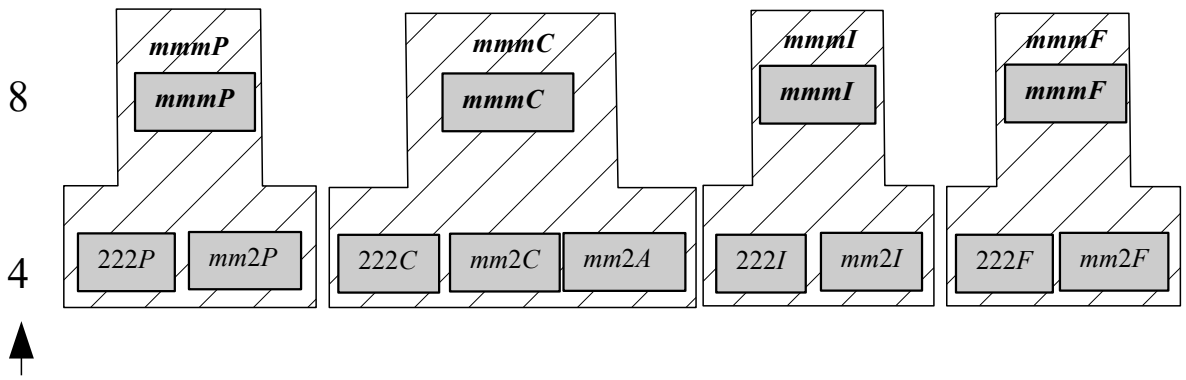
Ex. 2: Two crystals with holohedries $\bar{3}m$ and 6/*mmm* have lattices with two free parameters (*a* and *c*). Furthermore, $\bar{3}m$ is a subgroup of 6/*mmm* and thus the two crystals belong to the same crystal family (hexagonal).



Bravais classes

Bravais classes





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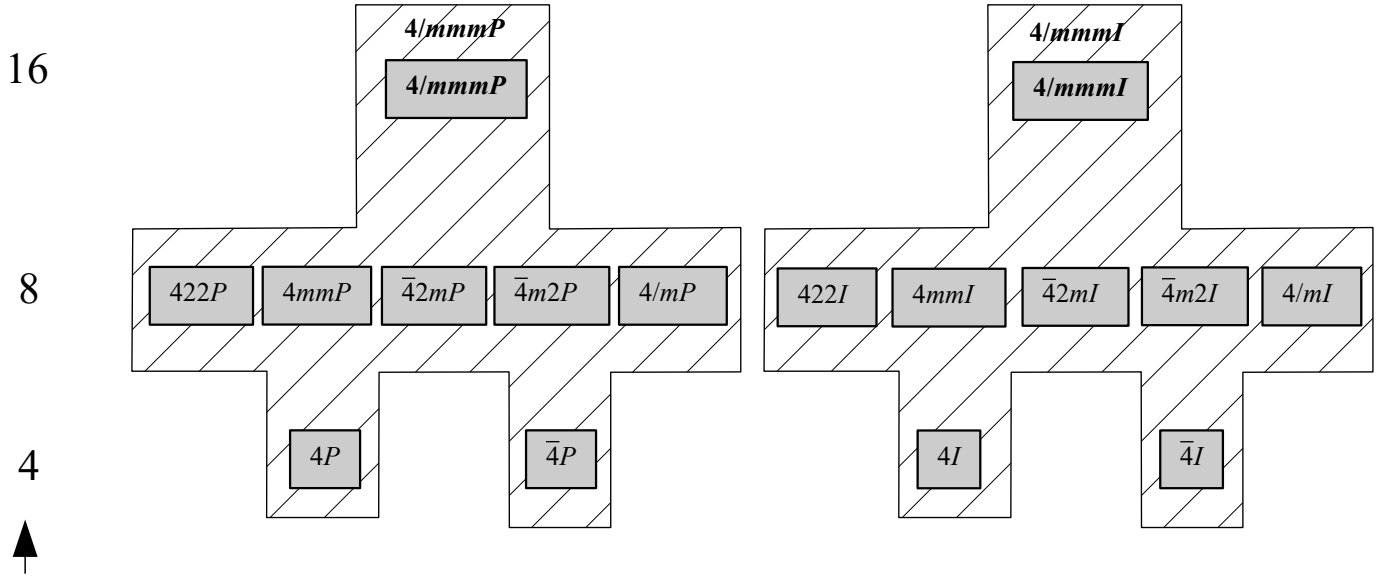
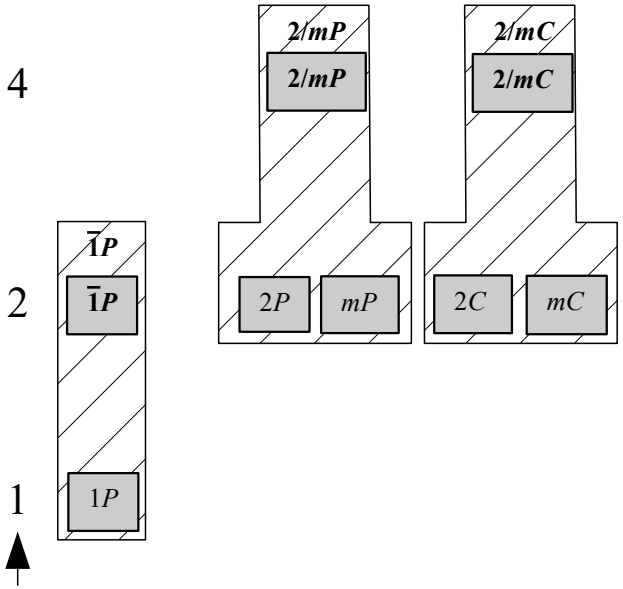
Bravais classes





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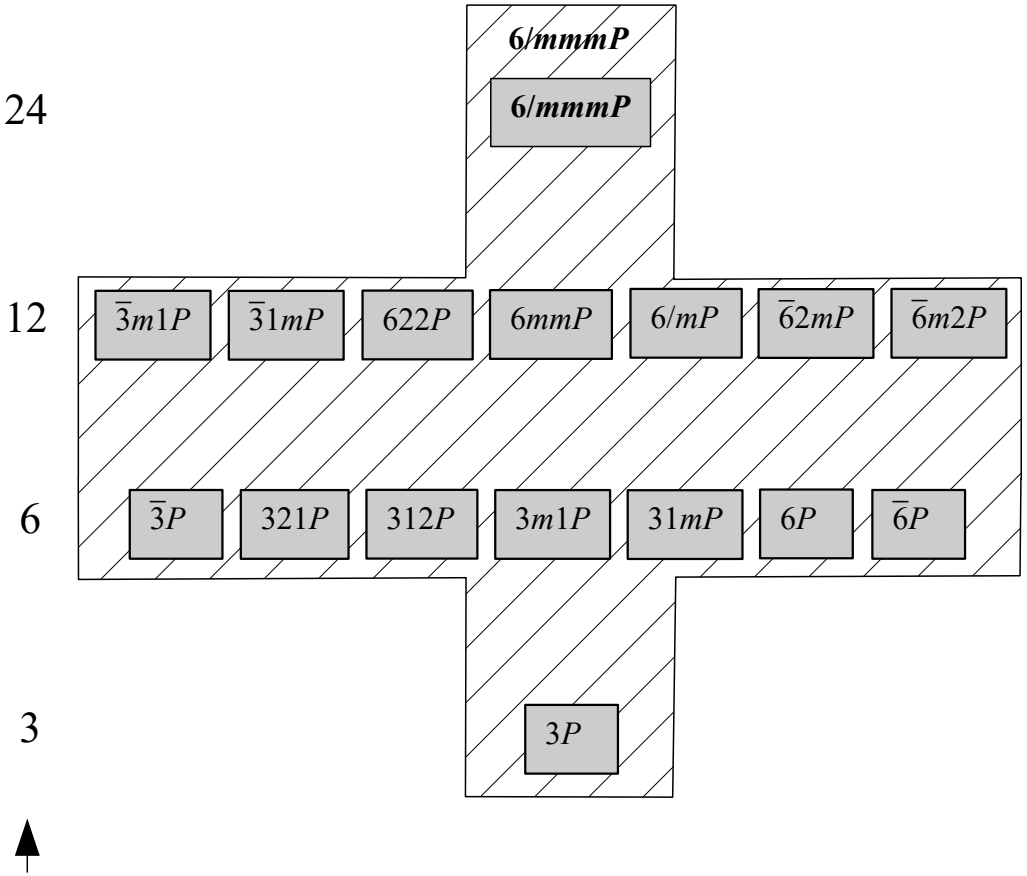
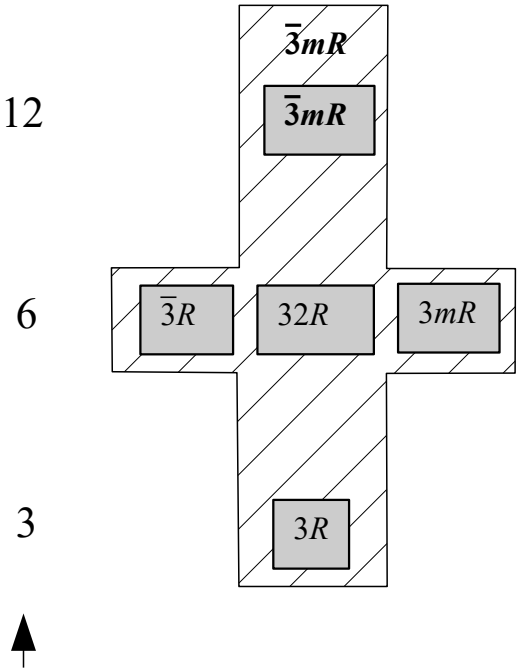
Bravais classes



Point group order



Point group order

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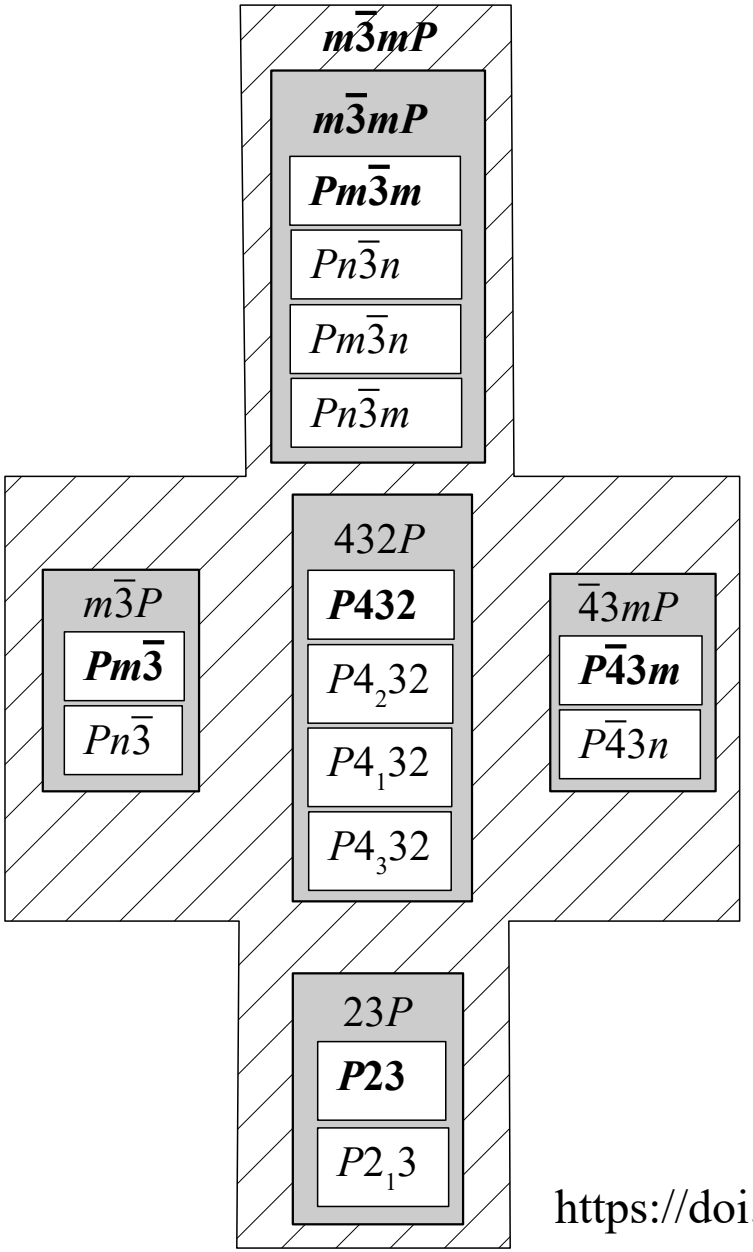
Bravais classes



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