Hierarchy of classification

Didactic material for the MaThCryst schools

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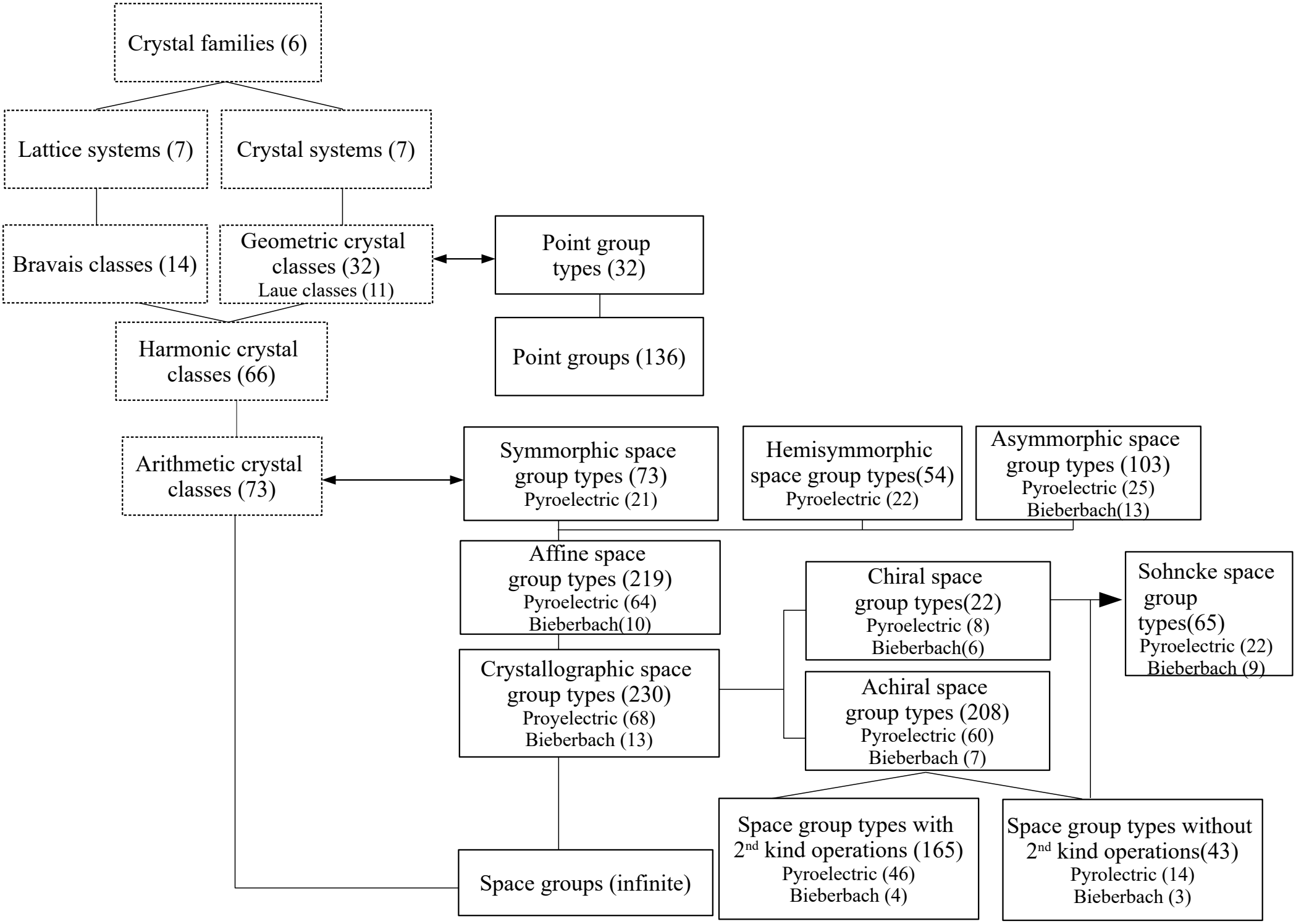
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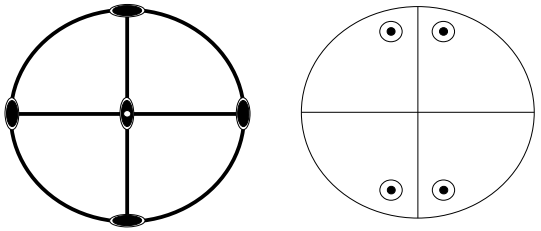
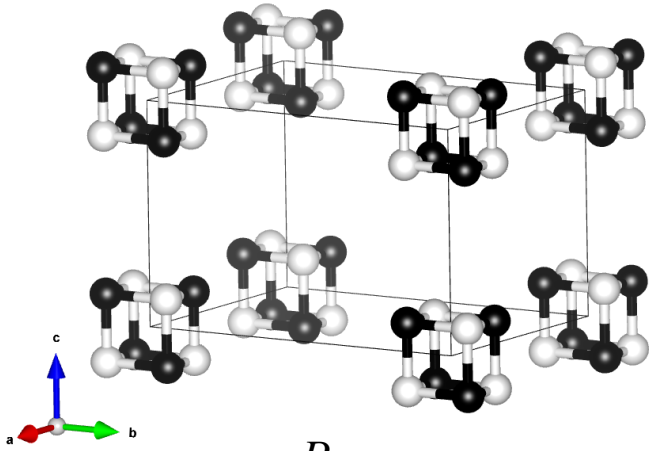
Point group



mmm

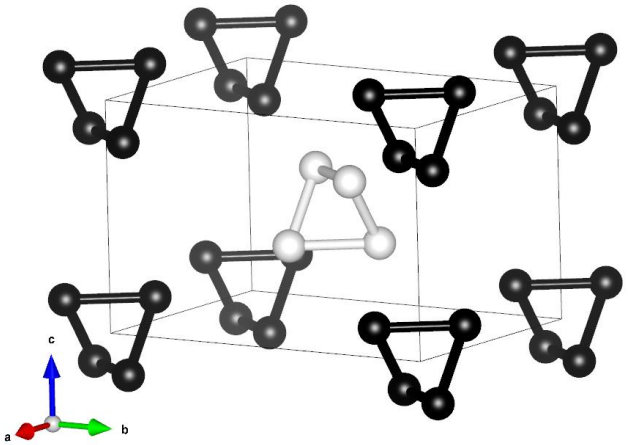
Symmorphic space group



Pmmm

site-symmetry group at the origin: *mmm*

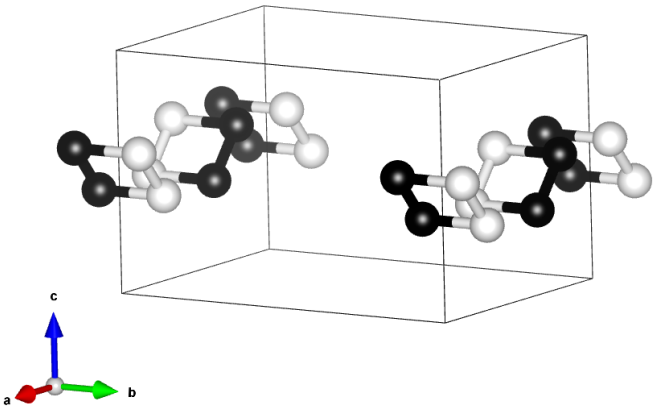
Hemisymmorphic space group



Pnnn

s.-s. group at the origin: 222

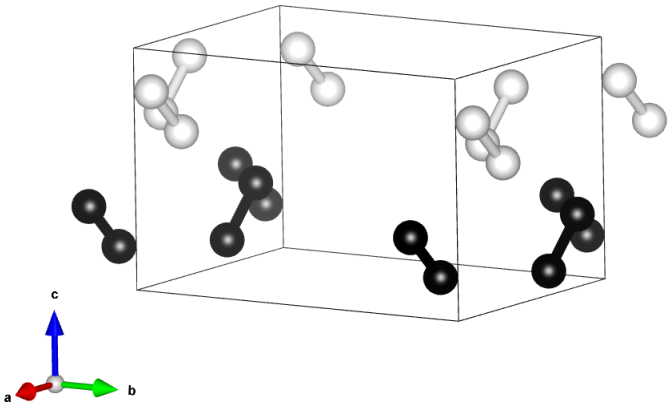
Asymmorphic space group



Pmma

s.-s. group at the origin: .2/*m*.

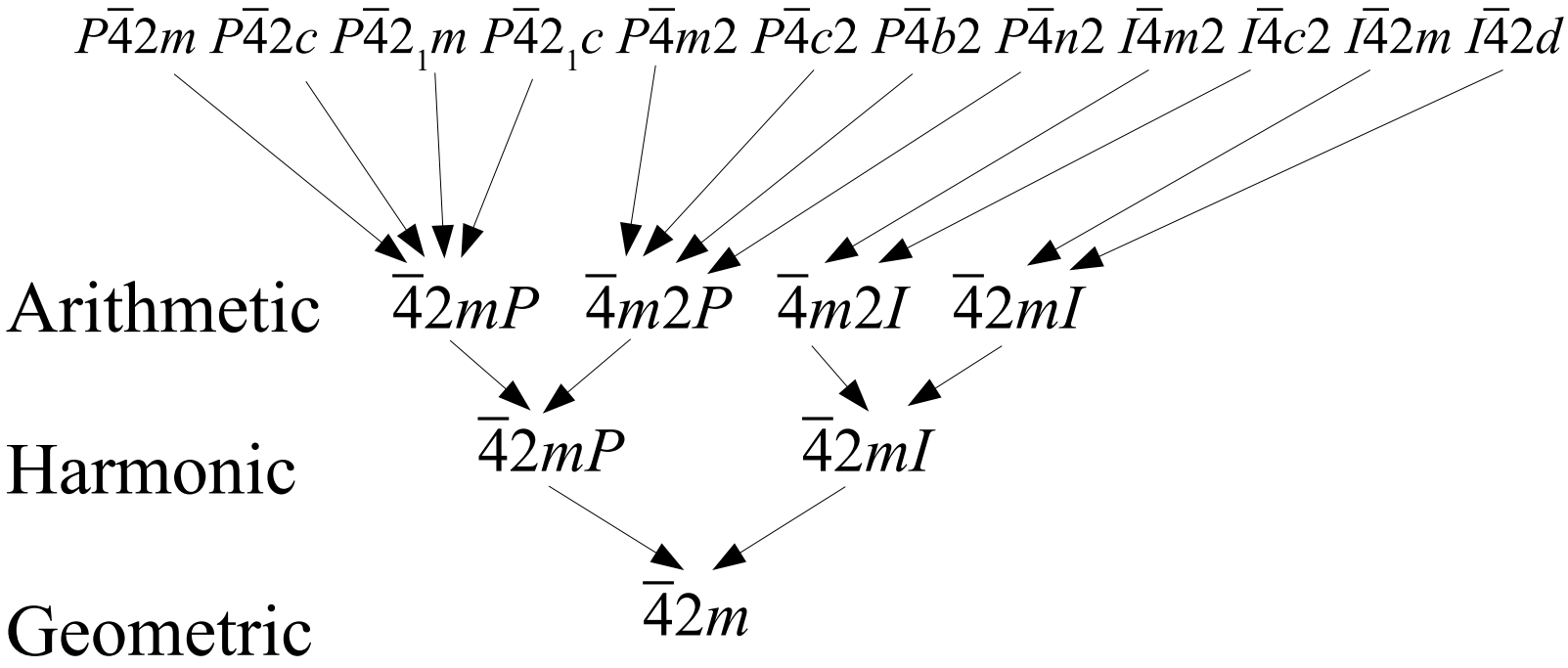
Asymmorphic space group



Pcca

s.-s. group at the origin: $\bar{1}$

Example of crystal classes hierarchy



A geometric crystal class which represents a full symmetry of a lattice is called a **holohedry**. Otherwise, a **merohedry**.

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Crystal systems

Space groups and crystal structures whose point group act of the same type of Bravais lattice belong to the same crystal system.

H, *G*: point groups $H \square G$

If *G* acts on lattice, then *H* to acts on the same lattice

If *H* acts on lattice, *G* does ***not necessarily*** act on the same lattice

Ex. 1: $m\bar{3}m$ (*G*) acts on *cP*, *cI*, *cF*. 23, *m*3, 432, $\bar{4}3m$ (*H*) act on the same lattices

23 (*H*) acts on *cP*, *cI*, *cF*. $m\bar{3}m$ (*G*) act on the same lattices. Etc.

Crystals with point groups $m\bar{3}m$, *m*3, 432, $\bar{4}3m$, 23
all belong to the cubic crystal system.

Ex. 2: 6/*mmm* (*G*) acts on *hP*. 6, $\bar{6}$, 6/*m*, 622, 6*mm*, $\bar{6}2m$, 3, $\bar{3}$, 32, 3*m*, $\bar{3}m$ (*H*) act on *hP* too.

$\bar{3}m$ (*H*) acts on *hP* and *hR* but 6/*mmm* (*G*) does not act on *hR*.

6 (*H*) acts on *hP*. 6/*mmm* (*G*) acts on *hP* too.

Crystals with point groups 6/*mmm*, 6, $\bar{6}$, 6/*m*, 622, 6*mm*, $\bar{6}2m$ all belong to the hexagonal crystal system. Those with point groups 3, 3, 32, 3*m*, $\bar{3}m$ belong to the trigonal crystal system.

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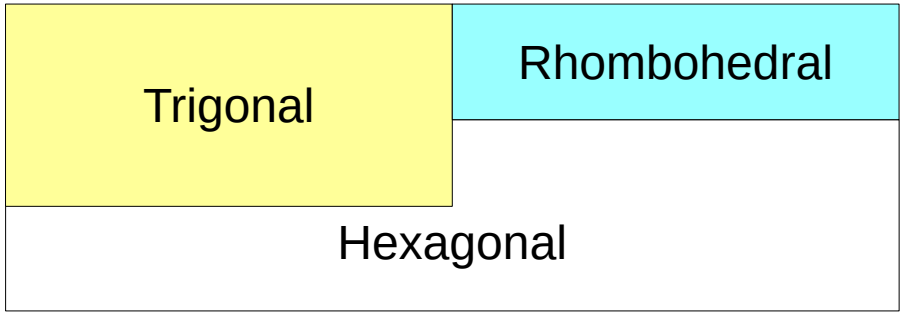
Lattice systems

Space groups and crystal structures which correspond to the same holohedry belong to the same lattice system.

Ex. 1: 23*P* and *m*3*P* correspond to the $m\bar{3}m$ holohedry and belong to the cubic lattice system.

Ex. 2: 32*P* and 622*P* correspond to the 6/*mmm* holohedry and belong to the hexagonal lattice system.

Ex. 3: 32*R* and 622*P* correspond to the different holohedries ($\bar{3}m$ and 6/*mmm*). 32*R* belong to the rhombohedral lattice system, whereas 622 belongs to the hexagonal lattice system.

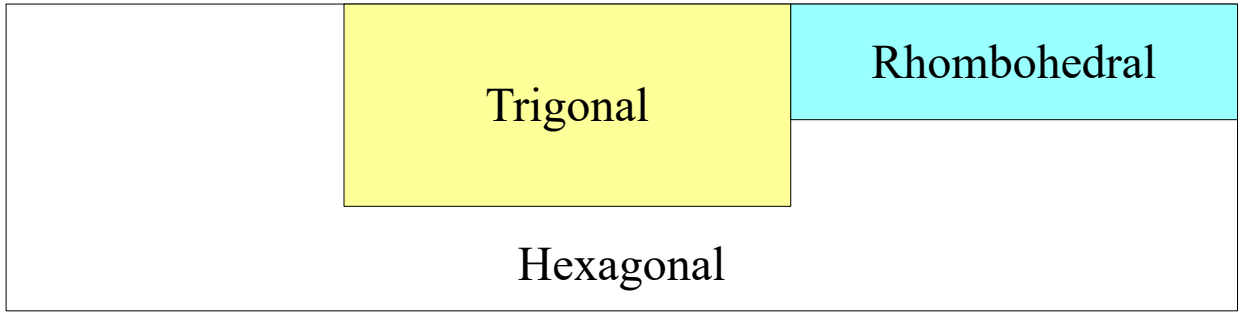


Crystal families

Space groups and crystal structures whose lattices have the same number of free parameters belong to the same crystal family if the corresponding point groups are in group-subgroup relation.

Ex. 1: Two crystals with holohedries 4/*mmm* and 6/*mmm* have lattices with two free parameters (*a* and *c*). However, 4/*mmm* and 6/*mmm* are not in group-subgroup relation and thus the two crystals belong to different crystal families (tetragonal and hexagonal).

Ex. 2: Two crystals with holohedries $\bar{3}m$ and 6/*mmm* have lattices with two free parameters (*a* and *c*). Furthermore, $\bar{3}m$ is a subgroup of 6/*mmm* and thus the two crystals belong to the same crystal family (hexagonal).



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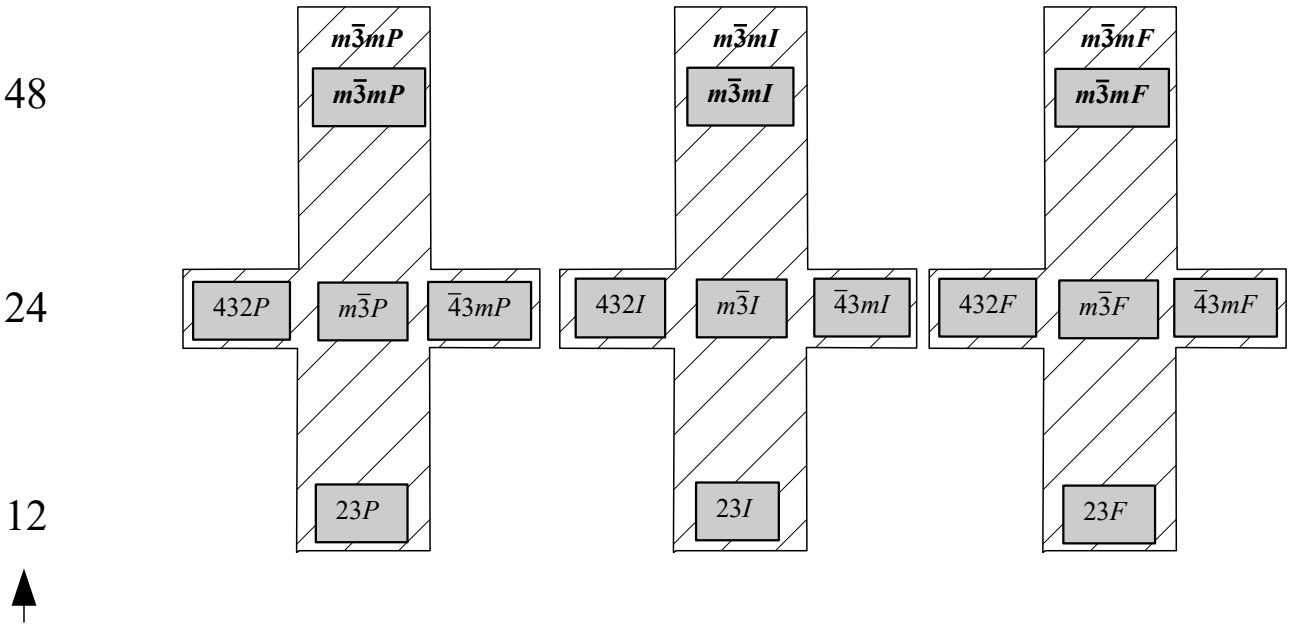
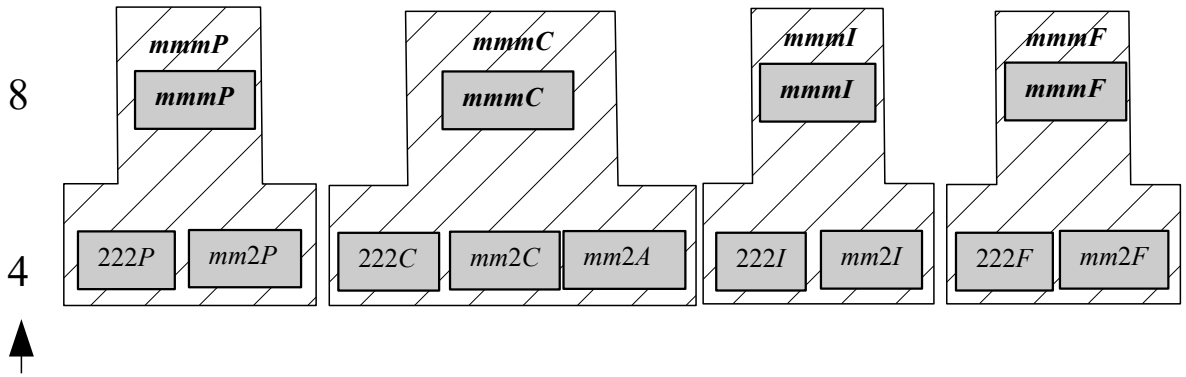
Bravais classes

Bravais classes





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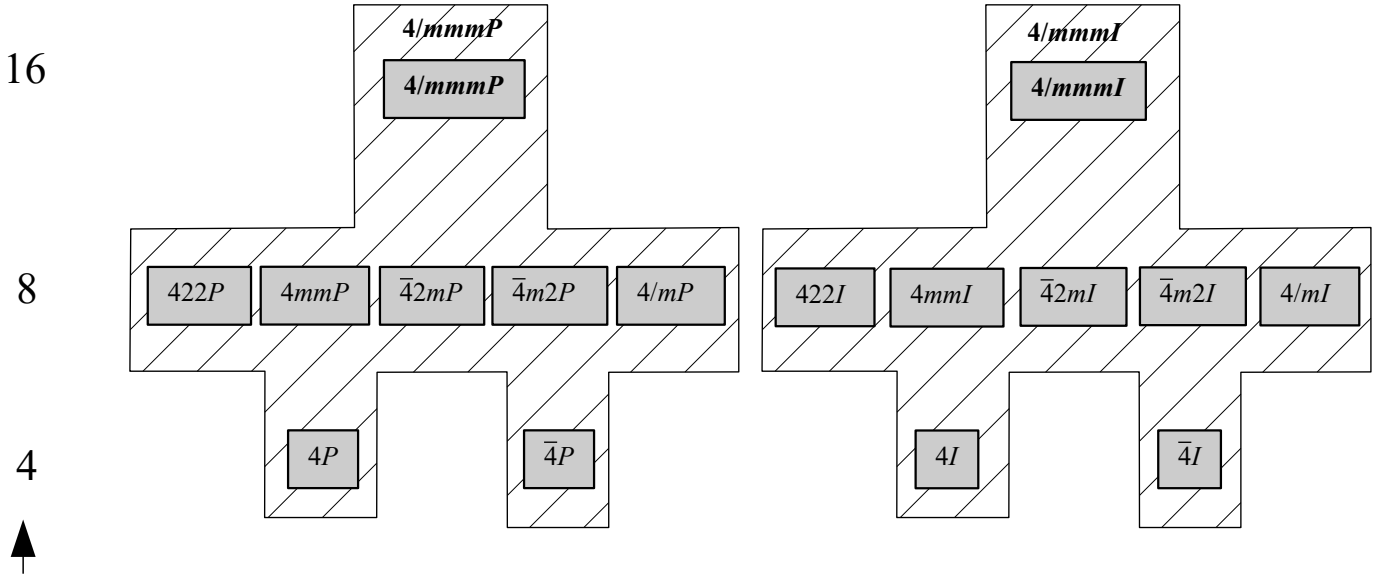
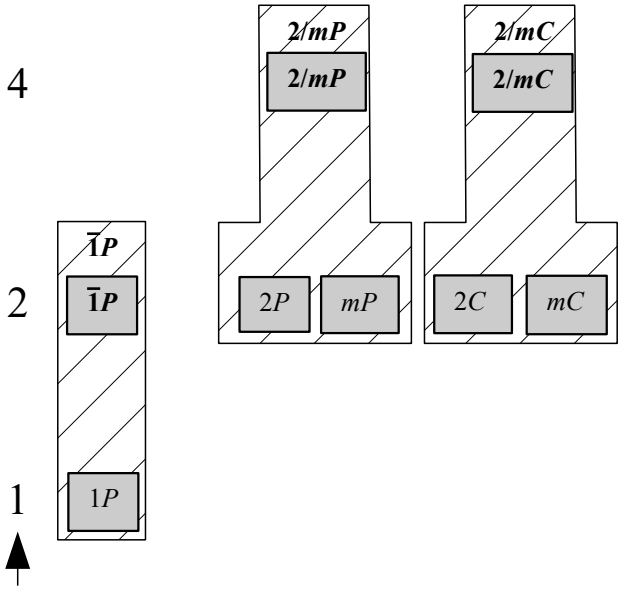
Bravais classes





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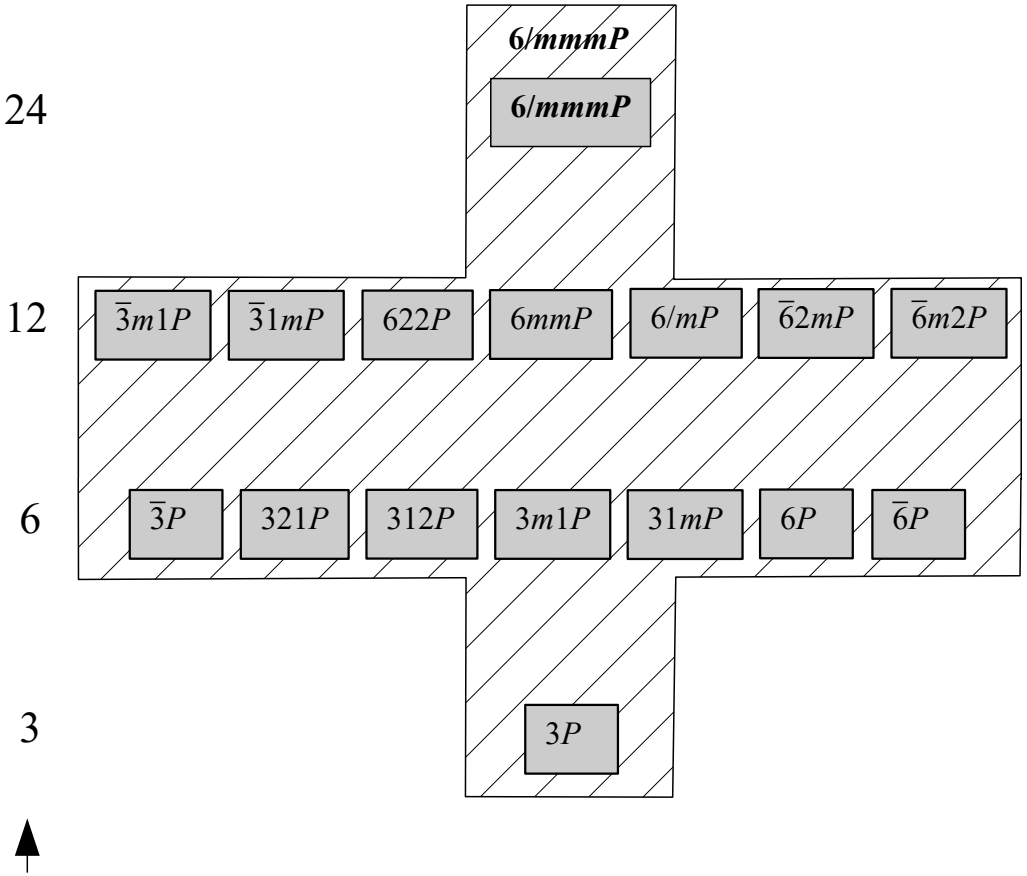
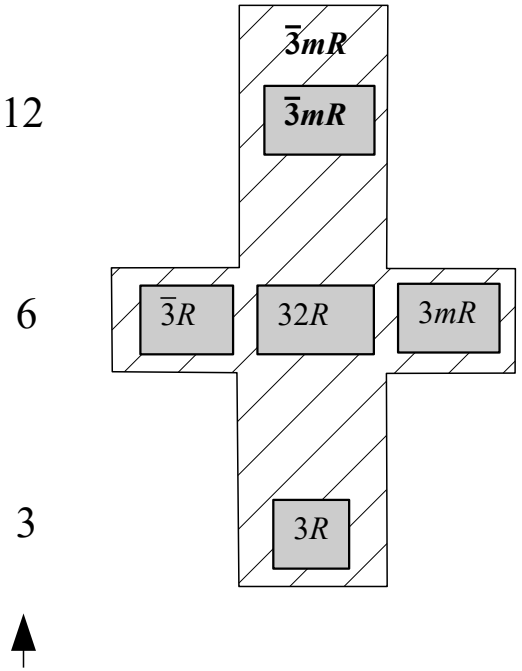
Bravais classes



Point group order



Point group order

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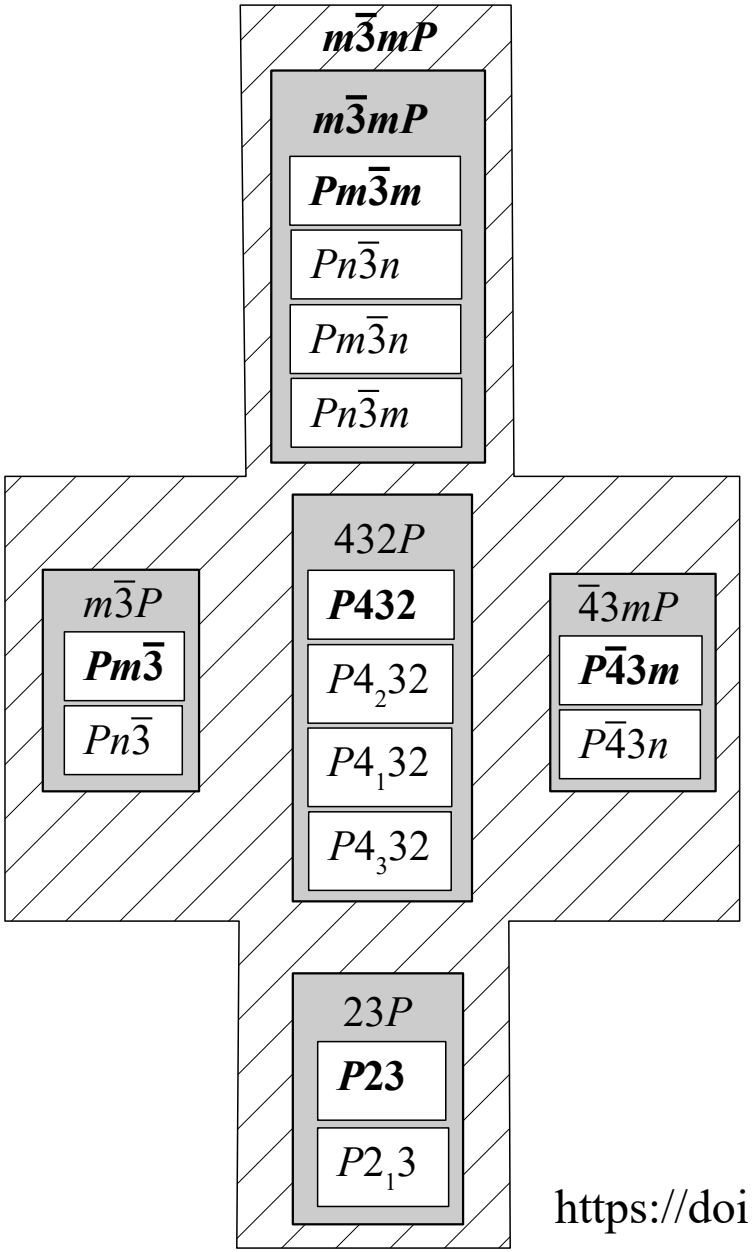
Bravais classes



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