

Screw axes and glide planes



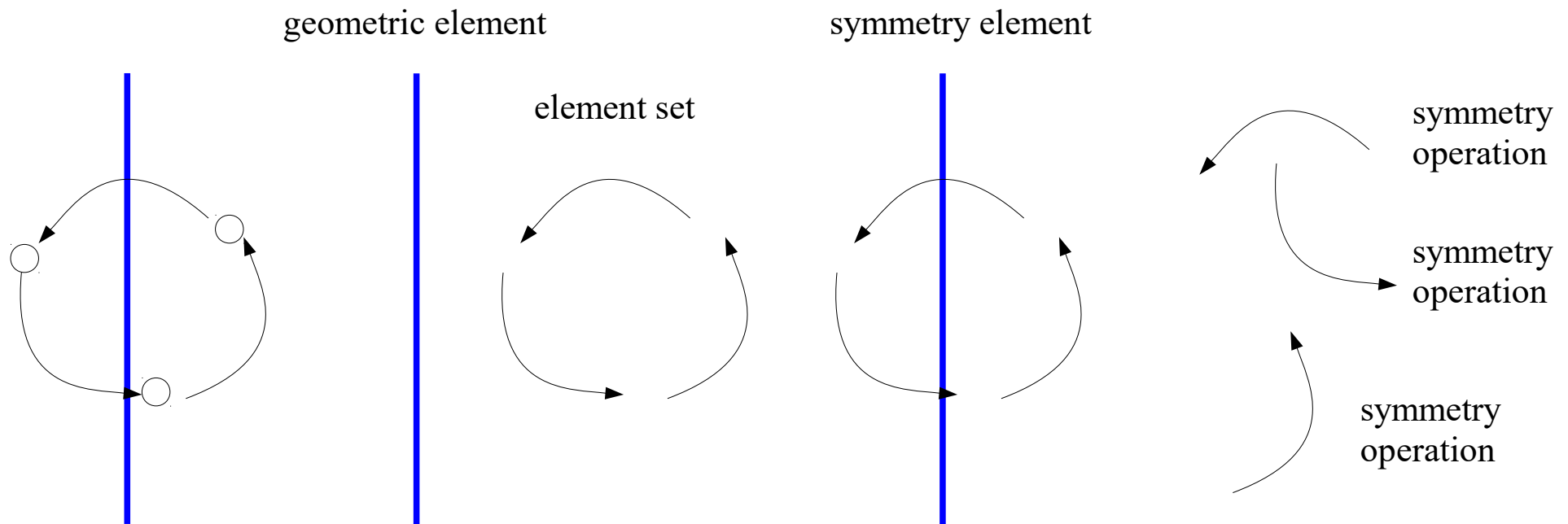
Didactic material for the MaThCryst schools

Massimo Nespolo, Université de Lorraine, France
massimo.nespolo@crm2.uhp-nancy.fr



Elements and operations

- **geometric element** : the point, line or plane left invariant by the symmetry operation.
- **symmetry element**: the geometric element defined above together with the set of operations (called **element set**) that leave it invariant.
- **symmetry operation** : an isometry that leaves invariant the object to which it is applied.



The operations that share a given geometric element differ by a lattice vector. The one characterized by the shortest vector is called **defining operation**.

Finding the geometric element

- Remove the intrinsic translation part (screw or glide component).
- Impose that the coordinates of the geometric element are fixed by the operation.

$$\left(\begin{array}{ccc|c} \bar{1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & \bar{1} & 1/2 \end{array} \right)$$

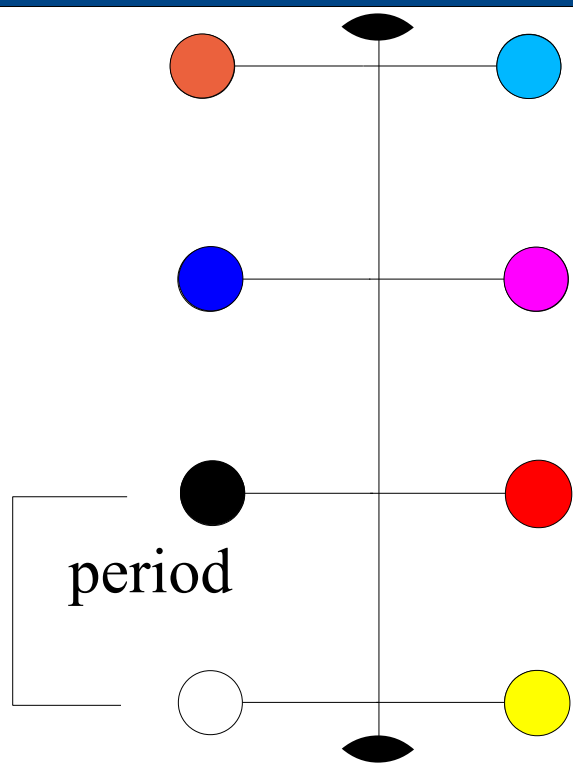
← Intrinsic part: screw component

← Location part: the element does not pass through the origin

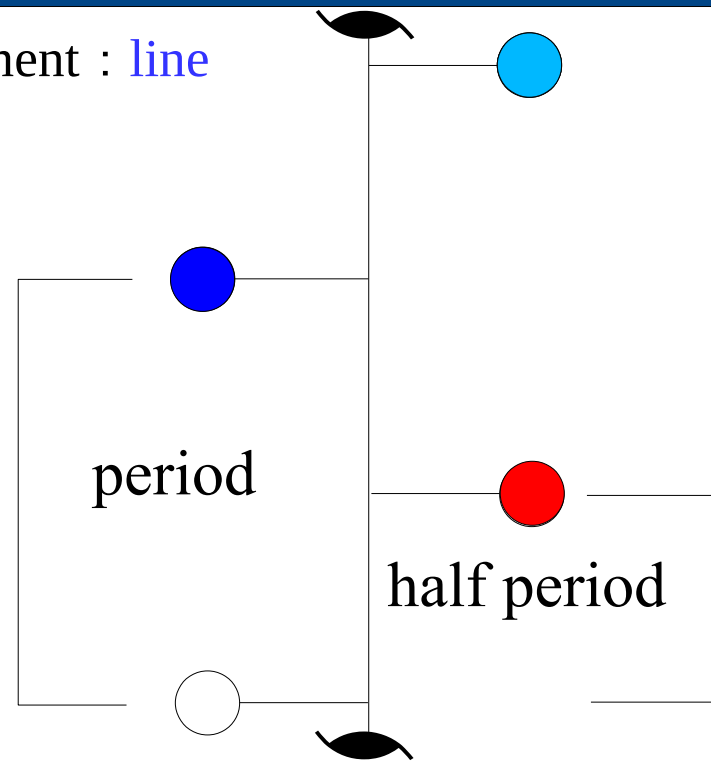
↑
Linear part: twofold rotation about [010]

$$\left(\begin{array}{ccc|c} \bar{1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \bar{1} & 1/2 \end{array} \right) \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \begin{cases} -u = u \\ v = v \\ -w + 1/2 = w \end{cases} \quad \begin{cases} u = 0 \\ \forall v \\ w = 1/4 \end{cases} \quad 0, y, 1/4$$

Screw axes n_p (screw component: p/n)



Geometric element : **line**



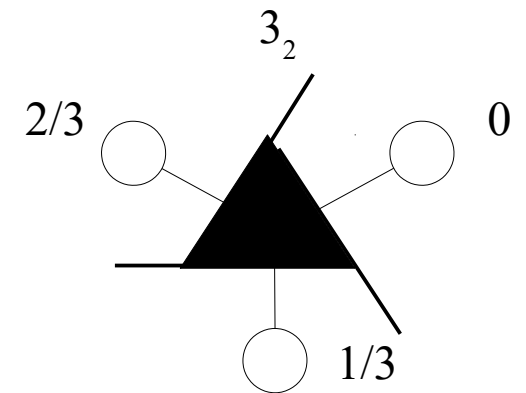
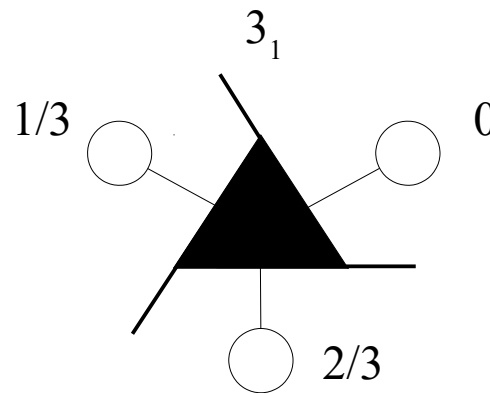
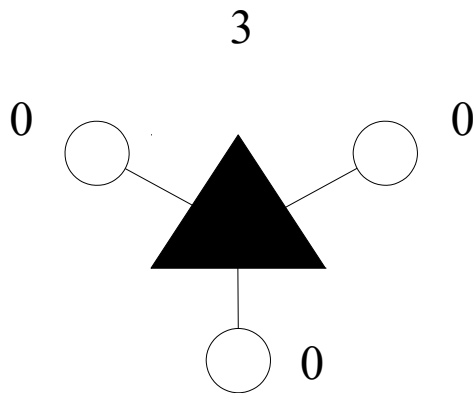
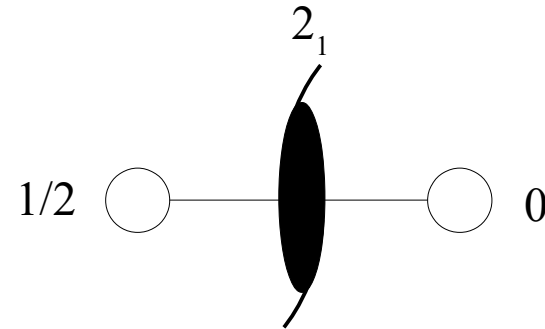
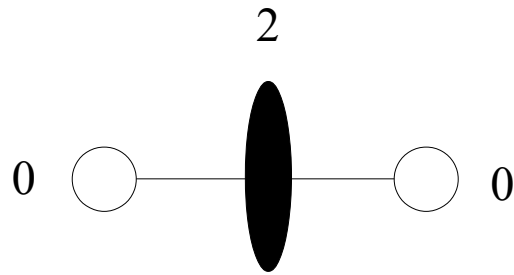
- → ● $2_0 = 2$
- → ● $t(001)$ ○ → ● 2_2
- → ● $t(002)$ ○ → ● 2_4
- → ● $t(003)$ ○ → ● 2_6

- → ● 2_1
- → ● $t(001)$
- → ● 2_3

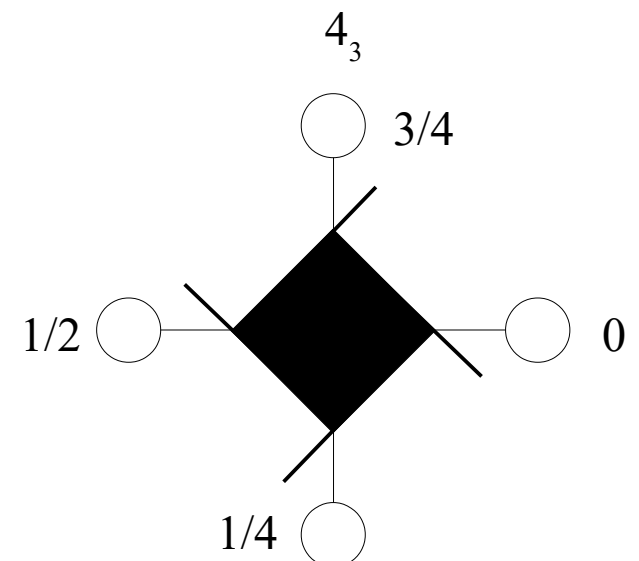
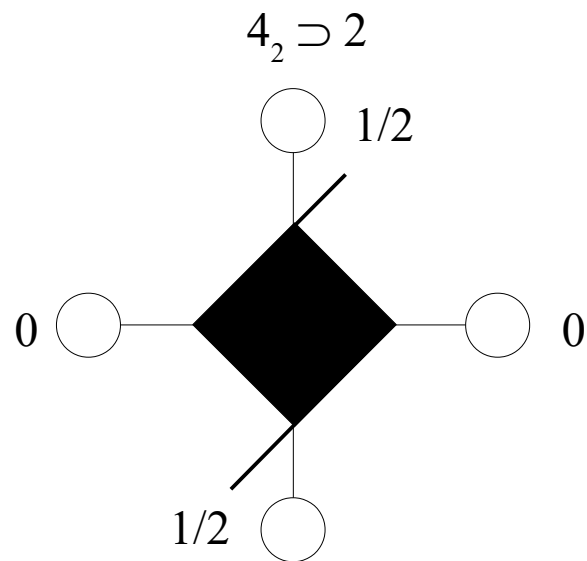
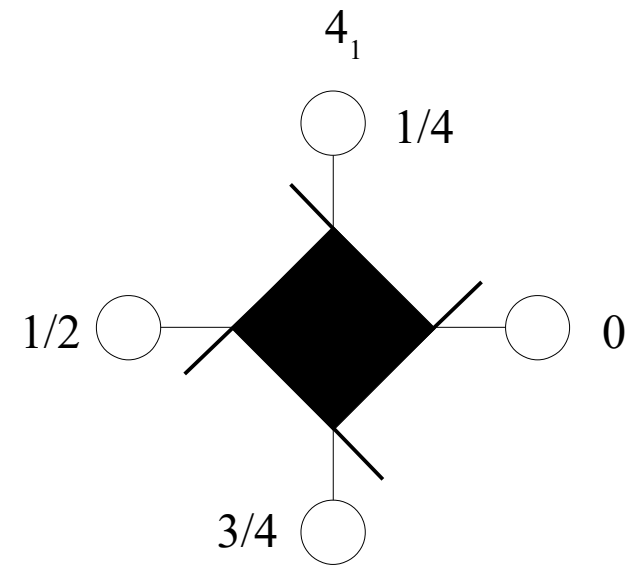
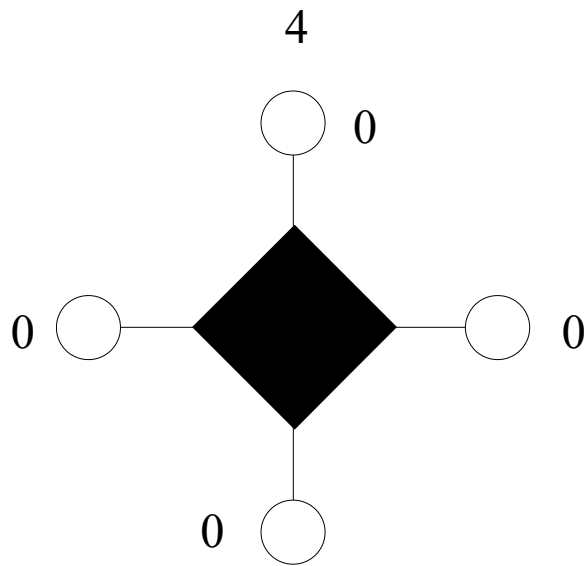
Defining operation : **twofold rotation**
 Symmetry element : **rotation axis**

Defining operation : **twofold screw rotation**
 Symmetry element : **screw axis**

Screw axes

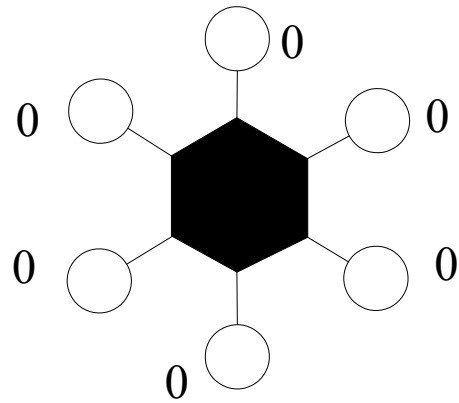


Screw axes

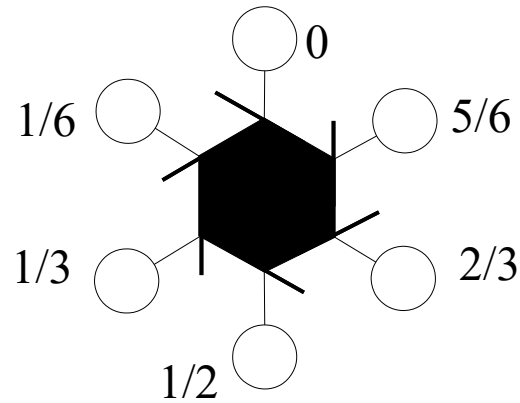


Screw axes

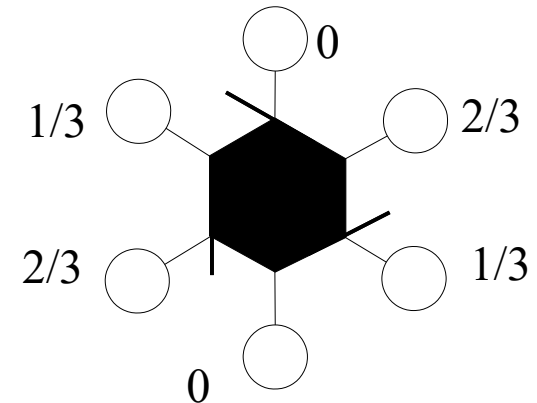
6



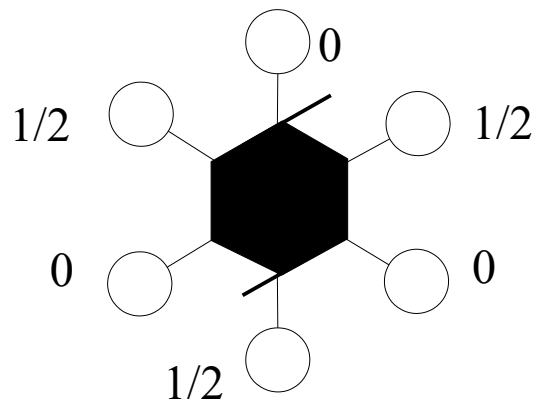
6_1



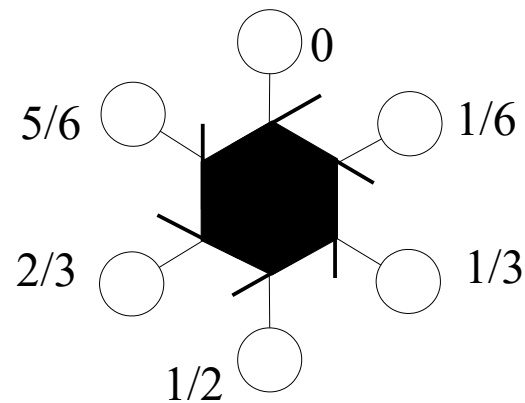
$6_2 \supset 2$



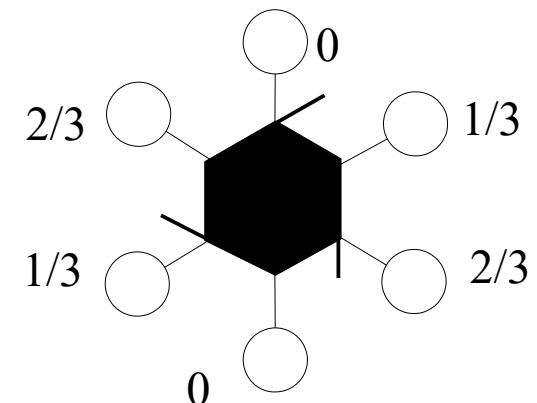
$6_3 \supset 3$



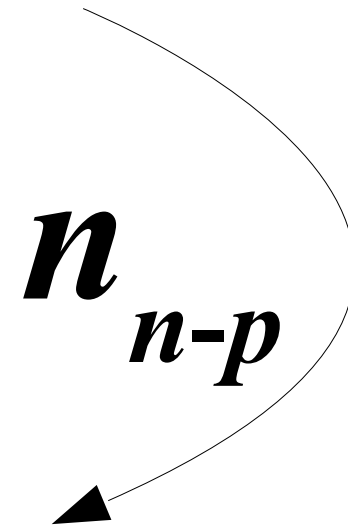
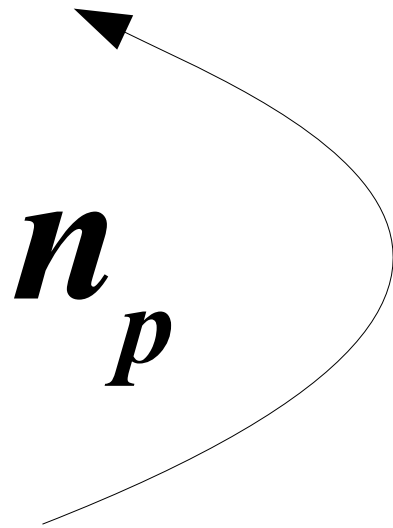
6_5



$6_4 \supset 2$



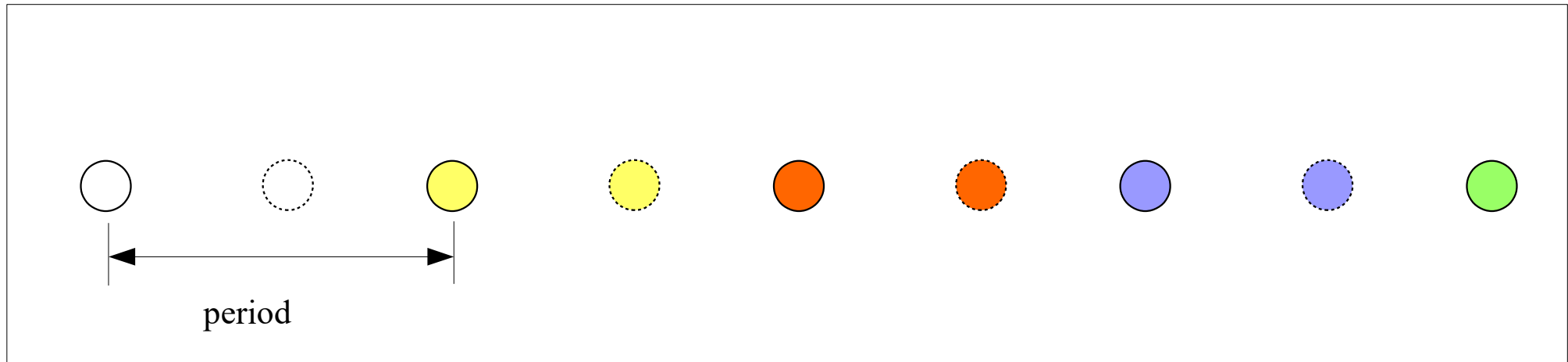
Screw axes



Glide planes g

Geometric element : **plane**

↑ [001]



○ → ○ $g (\frac{1}{2}, \frac{1}{2}, 0) x, x, z$

○ → ● $t(1, 1, 0)$

→ [110]

○ → ● $g (\frac{3}{2}, \frac{3}{2}, 0) x, x, z$

○ → ● $t(2, 2, 0)$

○ → ● $g (\frac{5}{2}, \frac{5}{2}, 0) x, x, z$

○ → ● $t(3, 3, 0)$

○ → ● $g (\frac{7}{2}, \frac{7}{2}, 0) x, x, z$

○ → ● $t(4, 4, 0)$

Special cases

$g (0, 0, 0): m$

$g (0, \frac{1}{2}, 0): b$

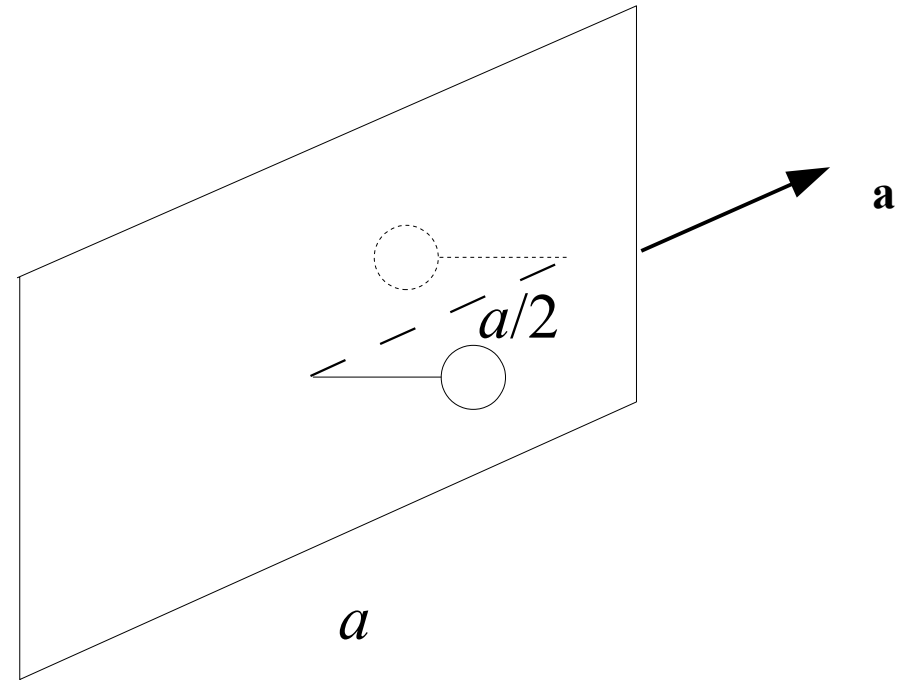
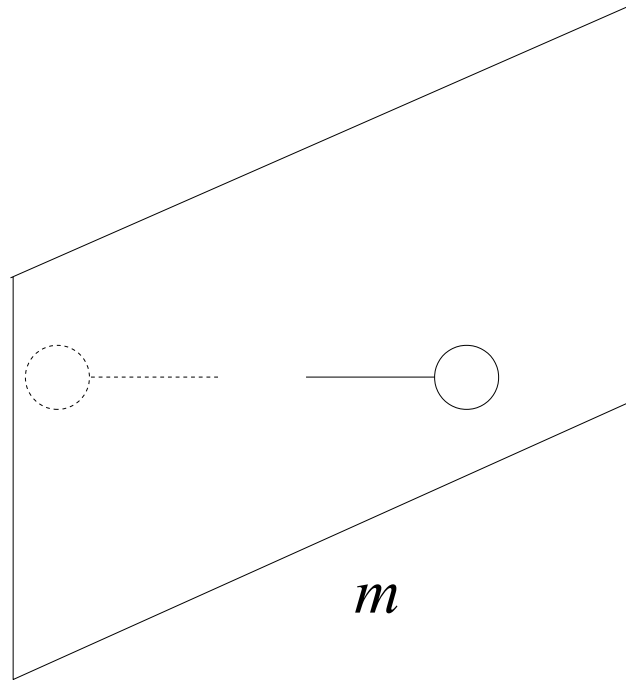
g (half-diagonal): n

$g (\frac{1}{2}, 0, 0): a$

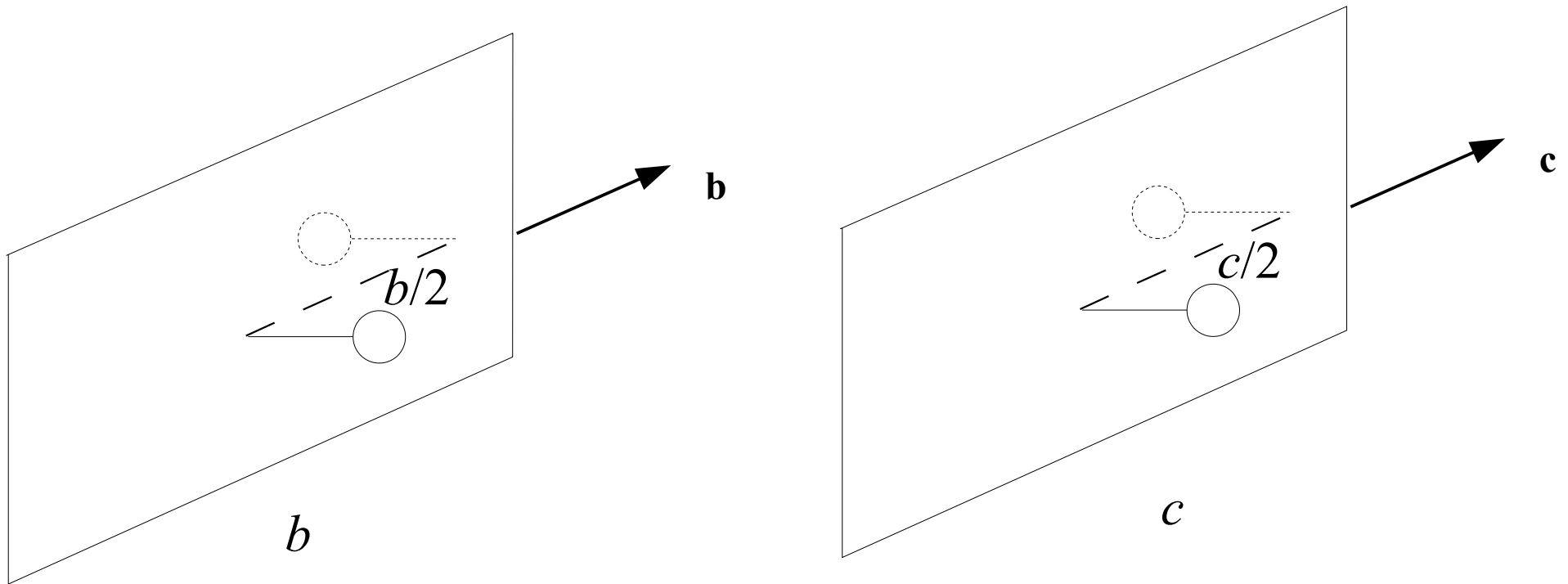
$g (0, 0, \frac{1}{2}): c$

g (quarter-diagonal): d

Glide planes

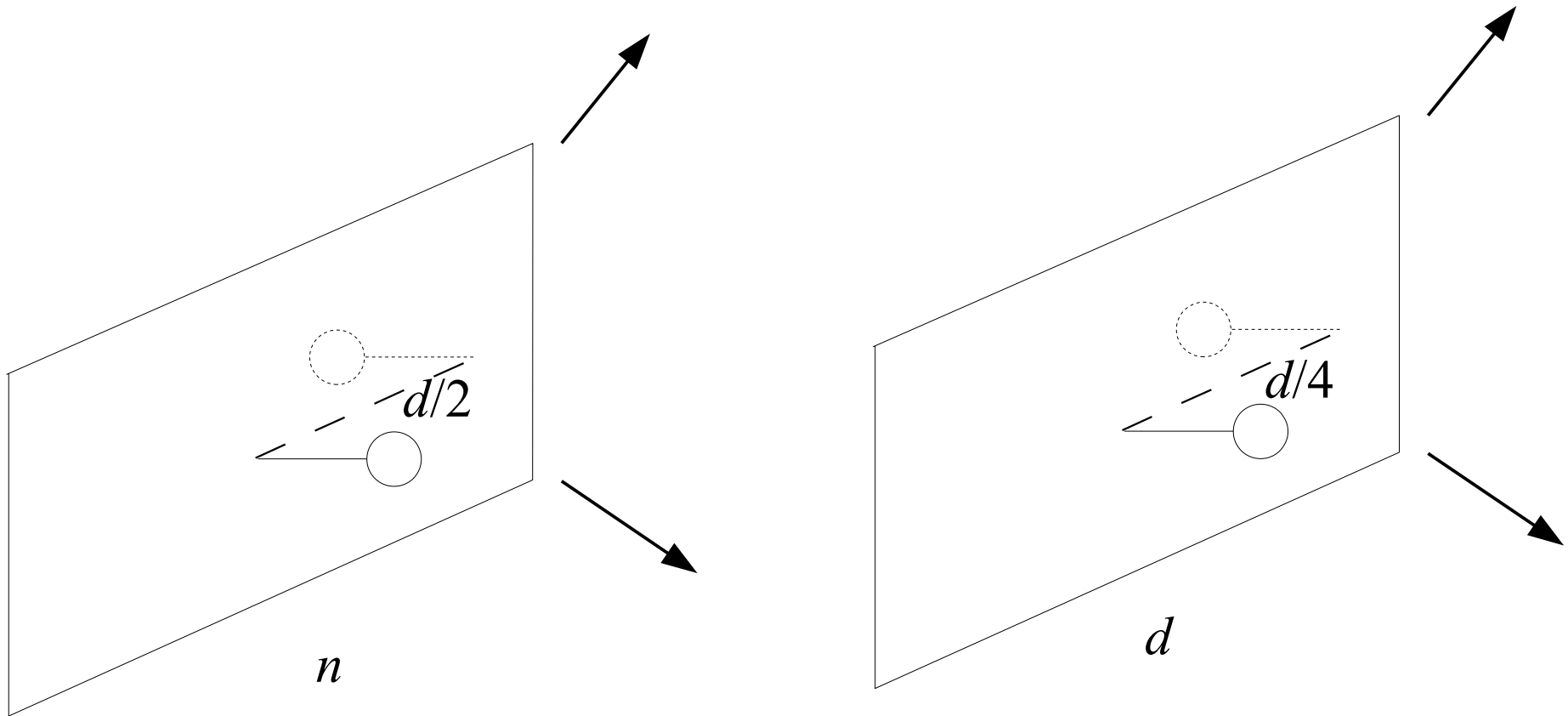


Glide planes



Diamond structure glide plane

Glide planes

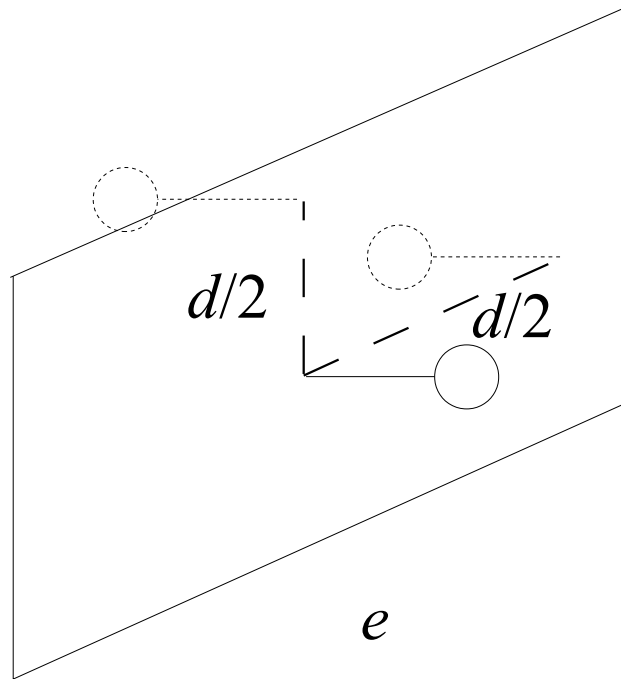


diagonal

Diamond structure glide plane

2D glide line; 3D glide plane with "unusual" glide

Glide planes



How can we have a glide of $\frac{1}{4}$ if the reflection is an operation of order 2 ?

If the unit cell is centred !

Vector centring the unit cell, with a norm $p/2$

